

Warm-Up: **Grab a compass and protractor!**

The circumference of a circle is 75.4 in.
Find the radius and diameter. Round
your answers to the nearest hundredths.

$$\begin{aligned} C &= \pi d \\ 75.4 &= \pi \cdot d \\ \frac{75.4}{\pi} & \\ 24.00 &= d \end{aligned} \quad \begin{aligned} C &= 2\pi r \\ r &= 12.00 \end{aligned} \quad \frac{75.4}{2\pi}$$



Circles in Coordinate Geometry

Objectives

- Use coordinates and the Pythagorean Theorem to derive the equation of a circle
- Complete the square to find the center and radius of a circle given by an equation

Circles in Coordinate Geometry

$$r = 4.5$$

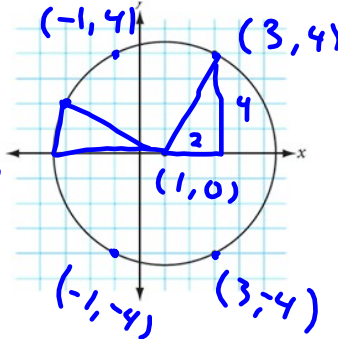
Launch

Find the radius of the circle.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(3 - 1)^2 + (4 - 0)^2}$$

$$d = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$$



$$2^2 + 4^2 = r^2$$

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Coordinate Geometry 10: Circles in Coordinat

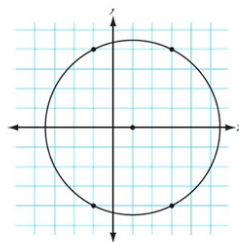


Circles in Coordinate Geometry

Launch

Find the radius of the circle.

$$\sqrt{20} \approx 4.47$$



Circles in Coordinate Geometry

Video game companies are continuously improving the gaming experience through better graphics and innovative gaming devices. For new games, software engineers must create and improve their codes. With respect to mathematics, anytime a software engineer wants to create a circle, she must graph the circle on the screen using a coordinate grid. What equation would she use? Let's use what we know about coordinate geometry to derive the equation of a circle that she would use.



INVESTIGATION

Equation of a Circle

In the last lesson you used the distance formula to find lengths of segments in a coordinate plane. The distance formula can also be used to find the equation of a circle. What is the equation of a circle with radius r and center (h, k) ? Use graph paper and compass or geometry software to investigate and make a conjecture.

Step 1 Given its center and radius, graph each circle.

Circle A : center = $(1, -2)$, $r = 8$

Circle B : center = $(0, 2)$, $r = 6$

Step 2 On each circle, select any point and label it (x, y) . Use the distance formula to write an equation for the distance from the center of the circle to (x, y) .

Step 3 Graph another circle with a different center and radius from Step 1. Repeat Step 2. Compare your equations with your group members. What do you notice?

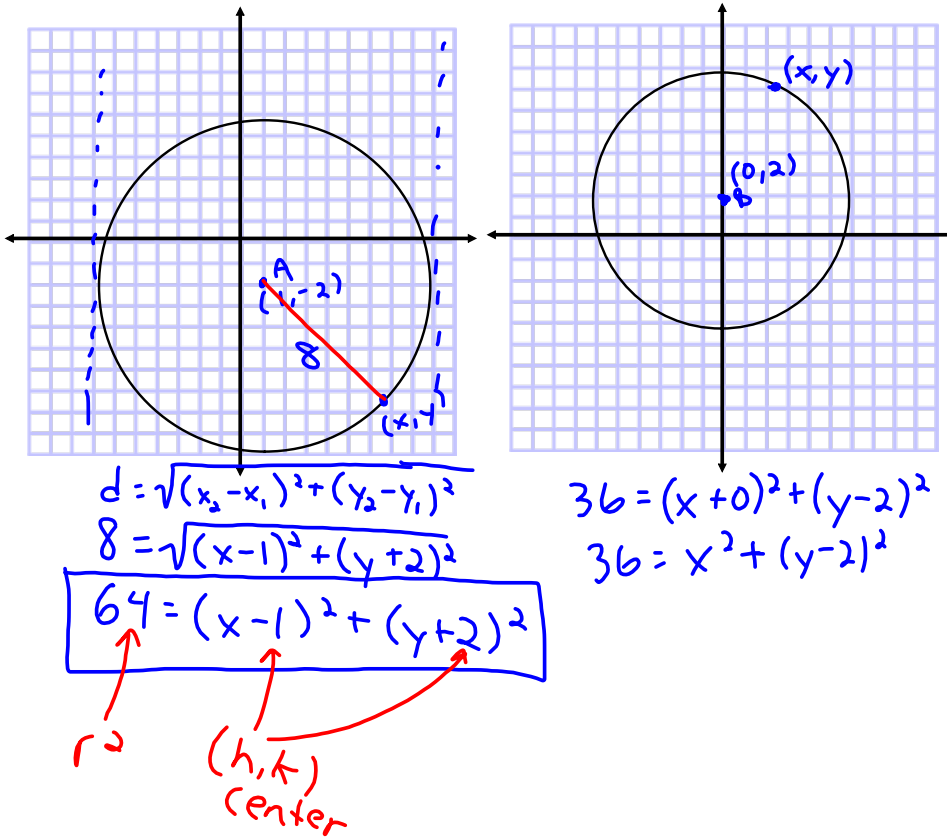
Step 4 Look for patterns, then copy and complete the conjecture.

Equation of a Circle

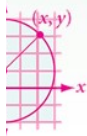
The equation of a circle with radius r and center (h, k) is: $(x - \underline{\quad})^2 + (y - \underline{\quad})^2 = \underline{\quad}$.

Circle A: center = (1, -2), r = 8

Circle B: center = (0, 2), r = 6



SOLUTION



le B

Step 1 Given its center and radius, graph each circle. Circle A: center = (1, -2), r = 8 Circle B: center = (0, 2), r = 6

Step 2 On each circle, select any point and label it (x, y). Use the distance formula to write an equation for the distance from the center of the circle to (x, y).

Circle A: $(x - 1)^2 + (y + 2)^2 = 64$; Circle B: $x^2 + (y - 2)^2 = 36$

Step 3 Graph another circle with a different center and radius from Step 1. Repeat Step 2. Compare your equations with your group members. What do you notice?

Answers will vary

Step 4 Look for patterns, then copy and complete the conjecture.

Equation of a Circle

The equation of a circle with radius r and center (h, k) is:

$$(x - h)^2 + (y - k)^2 = (r)^2.$$

EXAMPLE A

Write an equation for the circle with center $(5, 4)$ and radius 7 units.

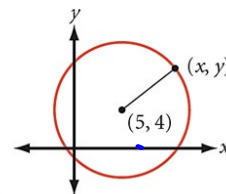
$$(x - 5)^2 + (y - 4)^2 = 49$$

SOLUTION

Let (x, y) represent any point on the circle. The distance from (x, y) to the circle's center, $(5, 4)$, is 7. Substitute this information in the distance formula.

$$(x - 5)^2 + (y - 4)^2 = 7^2$$

Substitute (x, y) for (x_2, y_2) . Substitute $(5, 4)$ for (x_1, y_1) . Substitute 7 as the distance.



So, the equation is $(x - 5)^2 + (y - 4)^2 = 7^2$.

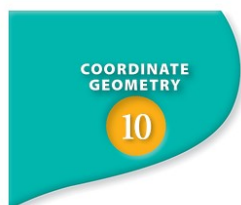


Circles in Coordinate Geometry

Summarize

$$x^2 = 9$$

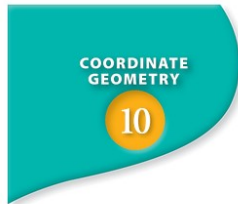
$$x = \pm 3$$



Circles in Coordinate Geometry

Summarize

- Should the equation include negative values of r , to be $r = \pm\sqrt{(x-h)^2 + (y-k)^2}$?



Circles in Coordinate Geometry

Summarize

- Should the equation include negative values of r , to be $r = \pm\sqrt{(x-h)^2 + (y-k)^2}$?

No; r is a radius, so it cannot be negative.



Circles in Coordinate Geometry

Extra Example

Find three points that lie on the circle defined by the equation

$$\cdot (4, 5) \quad (x-4)^2 + (y-1)^2 = 16.$$

Center: $(4, 1)$
radius: 4

$\cdot (0, 1)$ $(4, 1)$ $(8, 1)$

$\cdot (4, -3)$



Circles in Coordinate Geometry

Extra Example

ANSWER

Find three points that lie on the circle defined by the equation
 $(x - 4)^2 + (y - 1)^2 = 16$.

Answers will vary. Possible answers: (8, 1), (0, 1), (4, 5)



Circles in Coordinate Geometry

Closing Question

Write the equation of the circle with center (2, 3) that passes through the point (-1, 1).

$$\boxed{(x-2)^2 + (y-3)^2 = 13}$$

$$\text{radius} = \sqrt{(2+1)^2 + (3-1)^2}$$

$$= \sqrt{9+4}$$

$$r = \sqrt{13}$$



Circles in Coordinate Geometry

Closing Question

ANSWER

Write the equation of the circle with center (2, 3) that passes through the point (-1, 1).

$$(x - 2)^2 + (y - 3)^2 = 13$$

Homework:

Finish 9-1 Worksheet from yesterday

Workbook Pg. 78 1-4, 6, 7

