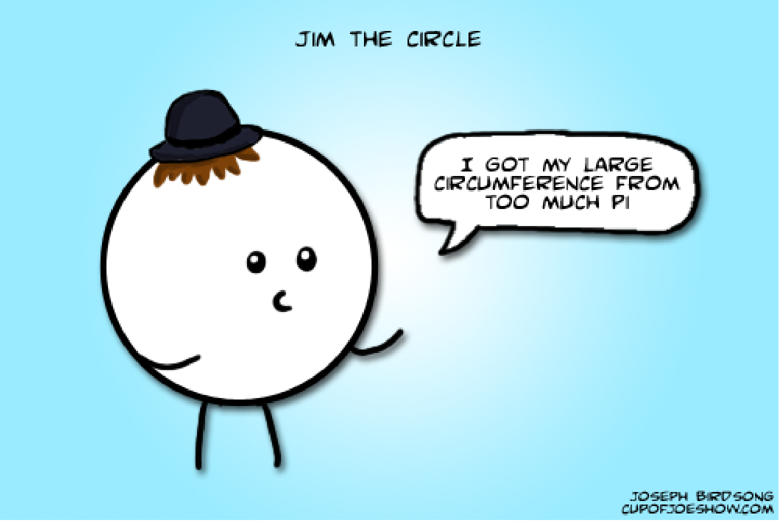
# Warm-Up:  Circle Social Media Task




Circle Task: Central Angles, Sectors          Name: ____________________

Dr. Bregy gave the high school students of District 300 a survey about the type of social media that they use the most often. Out of the total of 6157 high schoolers the most popular social media outlets were Facebook, Instagram, Twitter, Pinterest, and YouTube. The number of students using each outlet was 3325, 1108, 739, 616, and 369 respectively.

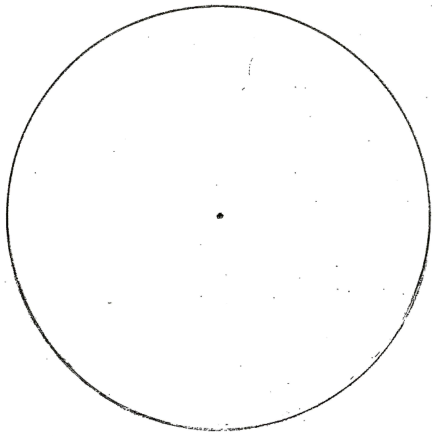
a. Find the percentage of students that prefer each social media outlet.

b. Find the measurement of the central angle representing each category.

c. Create a circle graph representing the data using a protractor.
   (Circle on the back)

d. Is the ARC LENGTH for Instragram congruent to any two combined social media arc lengths? Explain.

e. Measure the radius of your circle graph. Explain HOW can you find the ARC LENGTH of each sector (wedge) knowing just that information.



f. Find the Arc Length of the Instagram sector.

$d = 14.6\,cm$

$C = 14.6\pi$

$\sim 8.0 - 8.2$

$.18 \cdot 14.6\pi \approx 8.26$ cm

$\frac{65}{360} \cdot 14.6\pi$

## Circle Task 10.2

Key

a & b (round to whole number)

| Facebook | Instagram | Twitter | Pinterest | YouTube | TOTAL |
|---|---|---|---|---|---|
| 3325 | 1108 | 739 | 616 | 369 | 6157 |
| 54% | 18% | 12% | 10% | 6% | 100% |
| 194° | 65° | 43° | 36° | 22° | 100% of 360 |

d, e & f

Arc Length: ______________________

7.3cm

Radius: 73 mm

Circle Task: Central Angles, Sectors Name: Key

Dr. Bregy gave the high school students of District 300 a survey about the type of social media that they use the most often. Out of the total of 6157 high schoolers the most popular social media outlets were Facebook, Instagram, Twitter, Pinterest, and YouTube. The number of students using each outlet was 3325, 1108, 739, 616, and 369 respectively.

a. Find the percentage of students that prefer each social media outlet.

on pink sheet

b. Find the measurement of the central angle representing each category.

on pink sheet

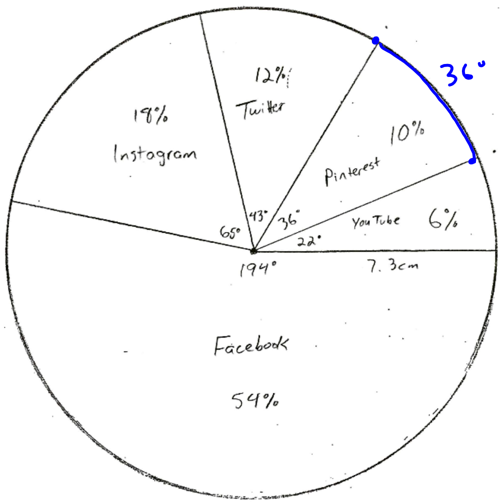
c. Create a circle graph representing the data using a protractor. (Circle on the back)

d. Is the ARC LENGTH for Instragram congruent to any two combined social media arc lengths? Explain.

YouTube and Twitter combined equals Instagram
6% 12% 18%

e. Measure the radius of your circle graph. Explain HOW can you find the ARC LENGTH of each sector (wedge) knowing just that information.

7.3 cm



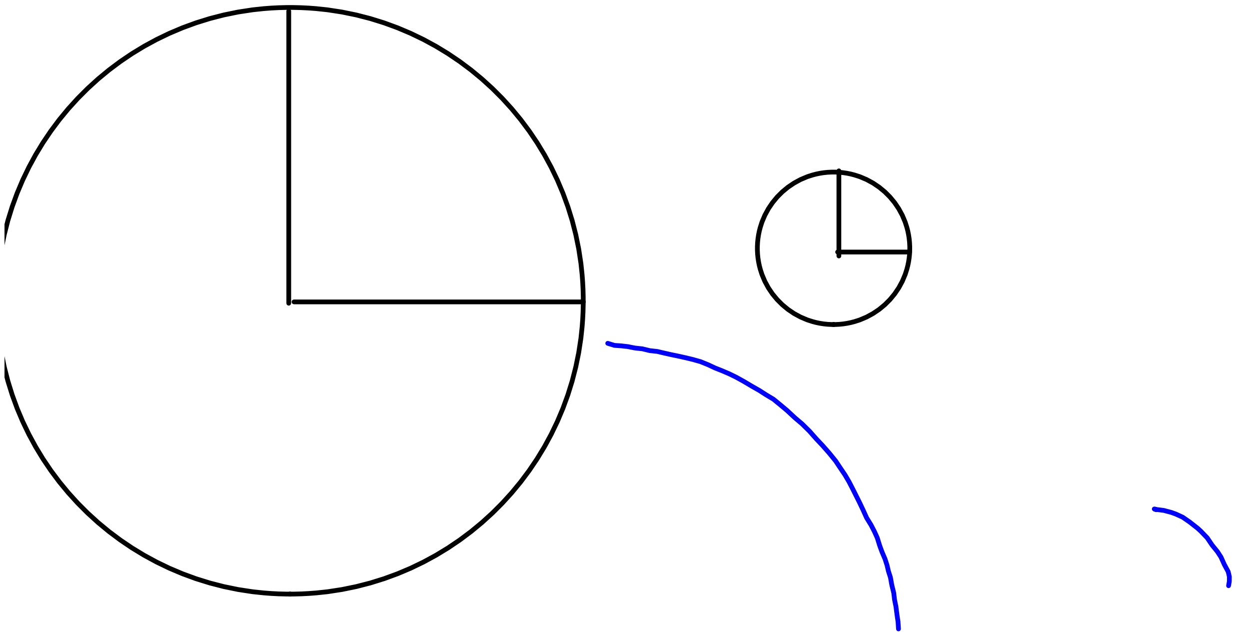

f. Find the Arc Length of the Instagram sector.

Circumference of entire circle $= 2\pi \cdot 7.3 = 14.6\pi$

Length of Instagram $=$ 18% of $14.6\pi = 2.628\pi$

or

$\frac{65}{360} \cdot 14.6\pi = 2.628\pi \approx 8.26$



LESSON 9.6

# Arc Length

| COMMON CORE STATE STANDARDS | | |
|---|---|---|
| Applied | Developed | Introduced |
| | G.C.5 | |

**G.C.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

LESSON 9.6

# Arc Length

## Objectives

- Discover a formula for finding the length of an arc of a circle
- Calculate arc length in radian measures
- Apply the formula for arc length





LESSON 9.6

Arc Length

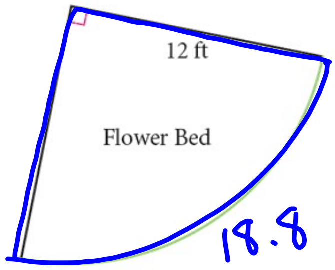
## Vocabulary

arc length

radian measure

# Launch

A landscaper designed a flowerbed in the shape of a quarter circle. If the radius of the circle is 12 ft, how much fencing does the landscaper need to enclose the flowerbed?




$$\text{Arc Length} = \frac{x}{360} \cdot 2\pi r$$

$$C = 2\pi \cdot 12 = 24\pi$$

$$\frac{90}{360} \cdot 24\pi = 6\pi \approx 18.8$$



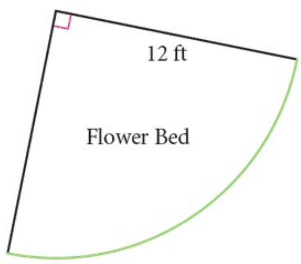
## 9.6 Arc Length

# Launch

A landscaper designed a flowerbed in the shape of a quarter circle. If the radius of the circle is 12 ft, how much fencing does the landscaper need to enclose the flowerbed?

$6\pi + 24$ ft $\approx 42.8$ ft

Exact Rounded

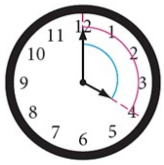


LESSON 9.6

# Arc Length

You have learned that the *measure* of a minor arc is equal to the measure of its central angle. On a clock, the measure of the arc from 12:00 to 4:00 is equal to the measure of the angle formed by the hour and minute hands. A circular clock is divided into 12 equal arcs, so the measure of each hour is $\frac{360°}{12}$, or 30°. The measure of the arc from 12:00 to 4:00 is four times 30°, or 120°.


Notice that because the minute hand is longer, the tip of the minute hand must travel farther than the tip of the hour hand even though they both move 120° from 12:00 to 4:00. So the arc *length* is different even though the arc *measure* is the same!




Let's take another look at the arc measure.

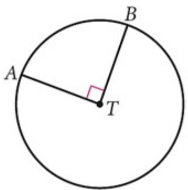


$\frac{150}{360} = \frac{5}{12}$

## EXAMPLE A

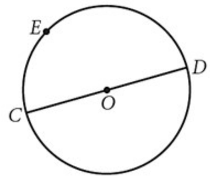
What fraction of its circle is each arc?

**a.** $\overset{\frown}{AB}$ is what fraction of circle *T*?



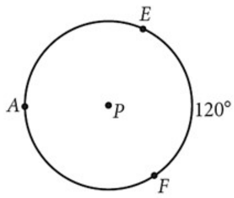

$\frac{90}{360} = \frac{1}{4}$

**b.** $\overset{\frown}{CED}$ is what fraction of circle *O*?




$\frac{1}{2}$

**c.** $\overset{\frown}{EF}$ is what fraction of circle *P*?




$\frac{1}{3}$

*SOLUTION*

In part a, you probably "just knew" that the arc is one-fourth of the circle because you have seen one-fourth of a circle so many times. Why is it one-fourth? The arc measure is 90°, a full circle measures 360°, and $\frac{90°}{360°} = \frac{1}{4}$. The arc in part b is half of the circle because $\frac{180°}{360°} = \frac{1}{2}$. In part c, you may or may not have recognized right away that the arc is one-third of the circle. The arc is one-third of the circle because $\frac{120°}{360°} = \frac{1}{3}$.





LESSON 9.6

# Arc Length

What do these fractions have to do with arc length? If you traveled halfway around a circle, you would cover $\frac{1}{2}$ of its perimeter, or circumference. If you went a quarter of the way around, you would travel $\frac{1}{4}$ of its circumference. The **arc length** is some fraction of the circumference of its circle.

The measure of an arc is calculated in units of degrees, but arc length is calculated in units of distance.

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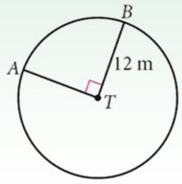
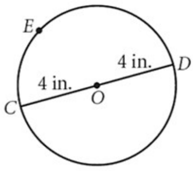
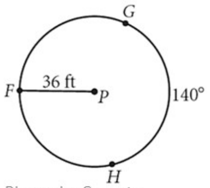
Lesson 9.6 Arc Length

# INVESTIGATION

## Finding the Arcs




In this investigation you will find a method for calculating the arc length.

**Step 1** For $\overset{\frown}{AB}$, $\overset{\frown}{CED}$, and $\overset{\frown}{GH}$, find what fraction of the circle each arc is.

**Step 2** Find the circumference of each circle.

**Step 3** Combine the results of Steps 1 and 2 to find the length of each arc.

**Step 4** Share your ideas for finding the length of an arc. Generalize this method for finding the length of any arc, and state it as a conjecture.

**Arc Length Conjecture** **C-87**

The length of an arc equals the ________ .





# INVESTIGATION SOLUTION

**Step 1** For $\overset{\frown}{AB}$, $\overset{\frown}{CED}$, and $\overset{\frown}{GH}$, find what fraction of the circle each arc is.

**Step 2** Find the circumference of each circle.

**Step 3** Combine the results of Steps 1 and 2 to find the length of each arc.

**Step 4** Share your ideas for finding the length of an arc. Generalize this method for finding the length of any arc, and state it as a conjecture.

**Steps 1–3:**

**Circle *T*** $\frac{90}{360} = \frac{1}{4}$, so $\overset{\frown}{AB}$ is $\frac{1}{4}$ of the circle. The radius is 12 meters, so the circumference is $24\pi$ meters. $\frac{1}{4}$ of $24\pi$ is $6\pi$, so the arc length of $\overset{\frown}{AB}$ is $6\pi$ meters.

**Circle *O*** $\frac{180}{360} = \frac{1}{2}$, so $\overset{\frown}{CED}$ is $\frac{1}{2}$ of the circle. The diameter is 8 inches, so the circumference is $8\pi$ inches. $\frac{1}{2}$ of $8\pi$ is $4\pi$, so the arc length of $\overset{\frown}{CED}$ is $4\pi$ inches.

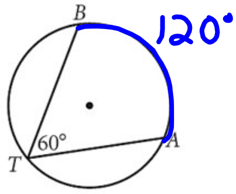
**Circle *P*** $\frac{140}{360} = \frac{7}{18}$, so $\overset{\frown}{GH}$ is $\frac{7}{18}$ of the circle. The radius is 36 feet, so the circumference is $72\pi$ feet. $\frac{7}{18}$ of $72\pi$ is $28\pi$, so the arc length of $\overset{\frown}{GH}$ is $28\pi$ feet.

**Arc Length Conjecture** **C-87**

The length of an arc equals the measure of the arc divided by 360° times the circumference.

## EXAMPLE B

If the radius of the circle is 24 cm and $m\angle BTA = 60°$, what is the length of $\widehat{AB}$?



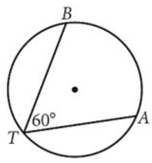

Exact + Rounded

$$\frac{x}{360} \cdot 2\pi \cdot 24$$

$$\boxed{\frac{120}{360} \cdot 48}\pi = 16\pi \text{ cm} \approx 50.3 \text{ cm}$$



## SOLUTION




$m\angle BTA = 60°$, so $m\widehat{AB} = 120°$ by the Inscribed Angle Conjecture. Then $\frac{120}{360} = \frac{1}{3}$, so the arc length is $\frac{1}{3}$ of the circumference, by the Arc Length Conjecture.

$$\text{arc length} = \frac{1}{3}C$$

$$= \frac{1}{3}(48\pi) \qquad \text{Substitute } 2\pi r \text{ for } C\text{, where } r = 24.$$

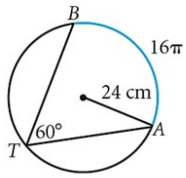
$$= 16\pi \qquad \text{Simplify.}$$

The arc length is $16\pi$ cm, or approximately 50.3 cm.

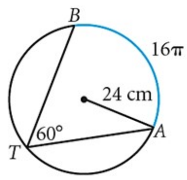
In this lesson, you learned the difference between the terms arc measure and arc length and learned how to find the arc length in degrees or linear units of measurement. Arc measure can also be measured in radians. A **radian measure** is calculated by dividing the length of the arc by its radius.

### *EXAMPLE C*

For Example B, find the radian measure of $\overset{\frown}{AB}$.






### *SOLUTION*

Using the definition of radian measure and the information from Example B, we can calculate the radian measure.

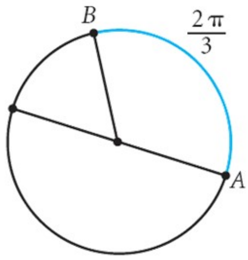
| | | |
|---|---|---|
| radian measure | $= \dfrac{\text{arc length}}{\text{radius}}$ | Definition of radian measure |
| | $= \dfrac{16\pi \text{ cm}}{24 \text{ cm}}$ | Substitute $16\pi$ for arc length and $r = 24$ |
| | $= \dfrac{2\pi}{3}$ radians | Simplify |

The radian measure of $\overset{\frown}{AB}$ is $\frac{2\pi}{3}$ radians. Since both the arc length and radius were in cm the units cancel, so there are no units associated with radians.

LESSON 9.6

# Arc Length

What do you notice about the radian measure in comparison to the semicircle? If we draw the diameter and the radius to point *B*, the relationship between the arc and the semicircle seems to be defined by the radian measure. What do you think the radian measure of the semicircle would be? How does the radius affect the radian measure? You will explore these ideas in the exercise set.






LESSON 9.6

# Arc Length

## Summarize

LESSON 9.6

# Arc Length

## Summarize

- What is the difference between arc measure and arc length?

- Why do radians have no units of measurement?





LESSON 9.6

# Arc Length

## Summarize

- What is the difference between arc measure and arc length?

  The arc measure is some fraction of the full circle, measured in degrees. The length of the arc is that same part of the circle's circumference, measured in units of distance.

- Why do radians have no units of measurement?

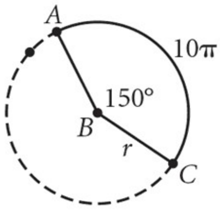
  Since radian measure is the ratio of arc length to the radius, or a ratio of lengths, the units cancel in the ratio.

LESSON 9.6

# Arc Length

## Extra Example

Find the radius.








LESSON 9.6

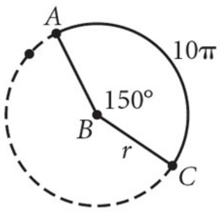
# Arc Length

## Extra Example

ANSWER

Find the radius.

$r = 12$

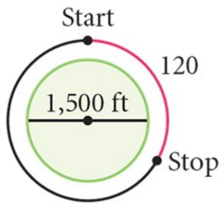


LESSON 9.6

# Arc Length

## Closing Question

Cedric was running along the circular path that has a diameter of 1500 feet shown below. The arc travelled before he had to stop was 120°. How far did he run in feet? How far did he run in radians?







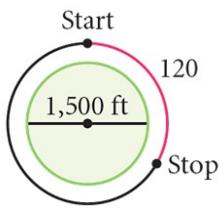

LESSON 9.6

# Arc Length

## Closing Question

ANSWER

Cedric was running along the circular path that has a diameter of 1500 feet shown below. The arc travelled before he had to stop was 120°. How far did he run in feet? How far did he run in radians?




In feet, the arc length is $\frac{120}{360} \cdot 1500\pi$ or $500\pi$ feet.

In radians, he ran $\frac{500\pi}{750}$, or $\frac{2\pi}{3}$ radians.

Homework: Workbook 9.6 (page 71)