

Mr. Simpson bought his nephew a bike with 14 inch diameter wheels for Christmas. Being the shrewd financial guru that he is, Mr. Simpson decided to build the bike himself instead of paying the store to do so. Below is the maiden voyage of the bike.

 <https://www.youtube.com/watch?v=4DO9LhQA1d4>

How could we figure out the distance this poor soul traveled while the wheel was still attached?

Section 9.5/9.6: Circumference and Arc Length

Learning Targets:

- I understand the relationship between the circumference and the radius of circle.
- I can apply the circumference formula to solve problems.
- I can calculate the length of the arc in a circle.

Exploration:

Step 1: Use a piece of yarn to measure the circumference of the round object that your group was given. To do this, wrap the yarn around the perimeter of the circular object that your group was given and measure the length of your yarn using your ruler. Record your measurement in the table below using millimeters.

Step 2: Measure the diameter of your circular object and record your measurement in the table below using millimeters. Remember, the diameter is the largest chord of the circle.

Step 3: Find the ratio of the circumference and the diameter by dividing the circumference by the diameter. Round your answer to the nearest hundredth. Record that result in the table below.

Step 4: Have a member of your group record your group's data in the table on the Smartboard.

	Circumference (mm)	Diameter (mm)	Ratio $\frac{C}{d}$
Mug	287	87	3.3
Tape	340	105	3.2
Marker	65	19	3.3

$$\text{Ratio} = \frac{C}{d}$$

Based on the results of the class, what do you notice about the ratio of the circumference and diameter?

Is there a number that you know of that is approximately equal to what you discovered about the ratio of the circumference and diameter? Take this information and write it as an equation.

Then, solve your equation for C.

$$\pi = \frac{C}{d} \rightarrow C = \pi d$$

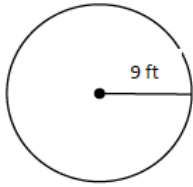
Circumference Conjecture

If C is the circumference and d is the diameter of a circle, then $C = \pi d$

Since $d = 2r$, where r is the radius of the circle, then $C = 2\pi r$

Find the circumference of each circle. Give the exact value and the decimal to the nearest tenth.

1.



$$r = 9$$

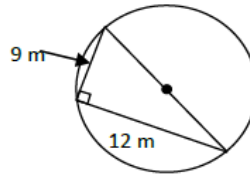
$$C = 2\pi \cdot 9$$

$$C = 18\pi$$

Exact Circumference = 18π ft

Decimal 56.5 ft

2.



Exact Circumference = 15π m

Decimal 47.1 m

$$9^2 + 12^2 = d^2$$

$$225 = d^2$$

$$15 = d$$

3. A child receives a new bike for his birthday which has 14 inch diameter wheels. His father puts it together for him and the child is able to briefly ride it before the front wheel falls off. If the tire rotated 11 times before the front wheel fell off, how many feet did the bike travel?

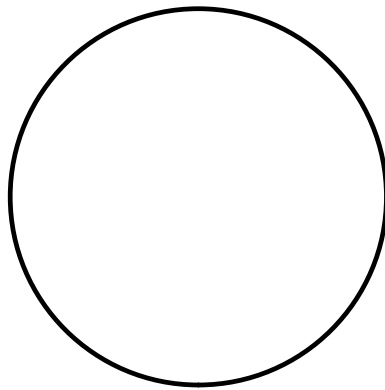
1 rotation = 1 circumference

$$C = \pi \cdot 14 = 14\pi \text{ in}$$

$$\text{distance} = 14\pi \cdot 11 = \boxed{154\pi \text{ in}} \text{ exact}$$

$$\approx 483.8 \text{ in}$$

$$\approx 40.3 \text{ ft}$$



Arc Length: The portion of the circumference of the circle described by an arc, measured in units of length.

Exploration:

Step 1: For \widehat{AB} , \widehat{CED} , and \widehat{GH} , find what fraction of the circle each arc is.

Step 2: Find the circumference of each circle.

Step 3: Combine the results in Steps 1 and 2 to find the length of each arc.

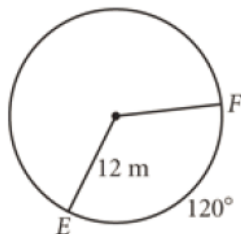
Circle	Fraction of the Circle	Circumference of the Circle	Arc Length
	$\frac{90}{360} = \frac{1}{4}$	$C = 2\pi \cdot 12$ $C = 24\pi$	$\frac{1}{4} \cdot 24\pi = 6\pi$ $6\pi \text{ m}$
	$\frac{180}{360} = \frac{1}{2}$	$C = \pi \cdot d$ $C = 8\pi$	$\frac{1}{2} \cdot 8\pi = 4\pi \text{ in}$
	$\frac{140}{360} = \frac{7}{18}$	$C = 2\pi \cdot 36$ $C = 72\pi$	$\frac{7}{18} \cdot 72\pi = 28\pi$ $28\pi \text{ ft}$
Radius = r Arc Measure = m	$\frac{m}{360}$	$C = \pi d$ $C = 2\pi r$	$\frac{m}{360} \cdot 2\pi r$ $\frac{m}{360} \cdot \pi d$

Arc Length Conjecture

If C is the circumference of a circle, then Arc Length = $\frac{m}{360}$ times the circumference (C) of the circle.

Answer each of the following. Round your answer to the nearest tenth if necessary.

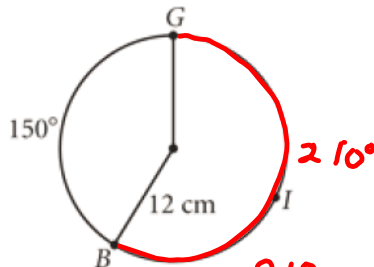
4. Find the length of \widehat{EF} .



$$\frac{120}{360} \cdot 2\pi \cdot 12$$

$$\boxed{25.1\text{m}}$$

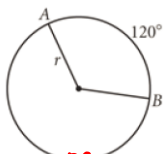
5. Find the length of $\widehat{B\overset{G}{I}}$.



$$\text{Length} = \frac{210}{360} \cdot 2\pi \cdot 12$$

$$\approx \boxed{44.0\text{cm}}$$

6. If the length of \widehat{AB} is 6π mm, find the radius.



$$\text{Length} = \frac{m}{360} \cdot 2\pi r$$

$$6\pi = \frac{120}{360} \cdot 2\pi r$$

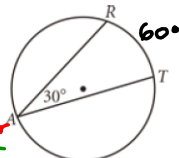
$$2160\pi = 120 \cdot 2\pi r$$

$$2160\pi = 240\pi r$$

$$9 = r$$

$$\boxed{9\text{mm}}$$

7. If the diameter of the circle is 18 ft, find the length of \widehat{RT} .



$$\text{Length} = \frac{60}{360} \cdot \pi \cdot 18$$

$$= \boxed{3\pi\text{ft}}$$

$$\approx 9.4\text{ft}$$

8. If a circle has an arc with measure of 160° and arc length of 12π inches, find the circle's diameter.
Work backwards

$$\text{length} = \frac{m}{360} \cdot \pi d$$

$$360 \cdot 12\pi = \frac{160}{360} \cdot \pi d \cdot 360$$

$$\frac{4320\pi}{160\pi} = \frac{160\pi d}{160\pi}$$

$$27 = d$$

$$\boxed{27\text{in}}$$

Homework:

Workbook Section 9.6 (page 71)

Warm-Up

Find the exact circumference of the circle below.

