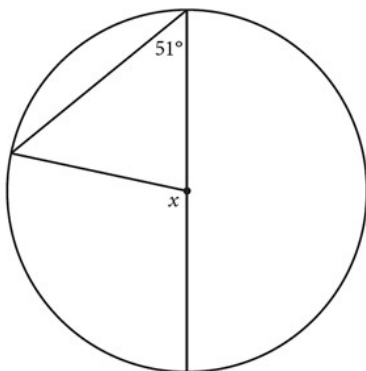


**Warm-up**

Find  $x$ .



hing

Lesson 9.3

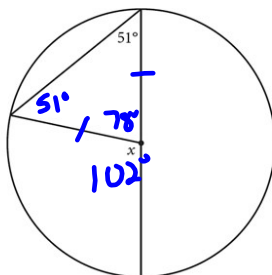
LESSON  
9.3

**Arcs and Angles**

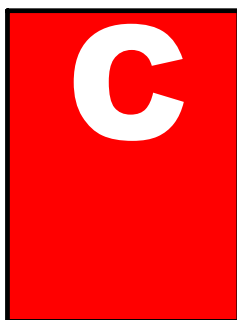
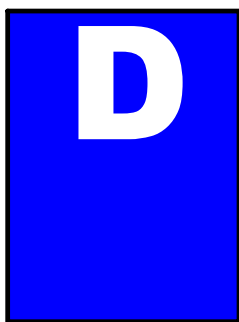
**Launch**

Find  $x$ .

$x = 102^\circ$



Decide in your group who will be the following:



## COMMON CORE STATE STANDARDS

Applied	Developed	Introduced
	G.C.2	
	G.C.3	

**G.C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

**G.C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.



## Arcs and Angles

### Objectives

- Discover relationships between an inscribed angle of the circle and its intercepted arc



## Arcs and Angles

### Vocabulary

cyclic quadrilateral  
secant

LESSON  
9.3

# Arcs and Angles

Many arches that you see in structures are semicircular, but Chinese builders long ago discovered that arches don't have to have this shape. The Zhaozhou bridge, shown below, was completed in 605 c.e. It is the world's first stone arched bridge in the shape of a minor arc, predating other minor-arc arches by about 800 years.



In this lesson you'll discover properties of arcs and the angles associated with them.

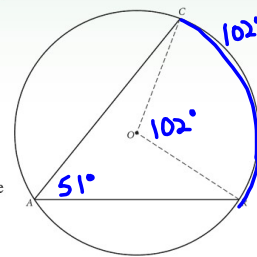


## INVESTIGATION 1

### Inscribed Angle Properties

**YOU WILL NEED:**  
compass,  
straightedge,  
protractor

In this investigation you will compare an inscribed angle and a central angle, both inscribed in the same arc. Refer to the diagram of circle  $O$ , with central angle  $COR$  and inscribed angle  $CAR$ .



**Step 1** Measure  $\angle COR$  with your protractor to find  $m\widehat{CR}$ , the intercepted arc. Measure  $\angle CAR$ . How does  $m\angle CAR$  compare with  $m\widehat{CR}$ ?

**Step 2** Construct a circle of your own with an inscribed angle. Draw and measure the central angle that intercepts the same arc. What is the measure of the inscribed angle? How do the two measures compare?

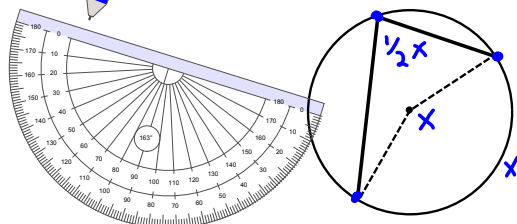
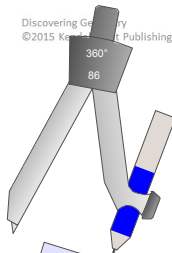
**Step 3** Share your results with others near you. Copy and complete the conjecture.

### Inscribed Angle Conjecture

C-81

The measure of an angle inscribed in a circle is half the measure of the intercepted arc.

Discovering Geometry  
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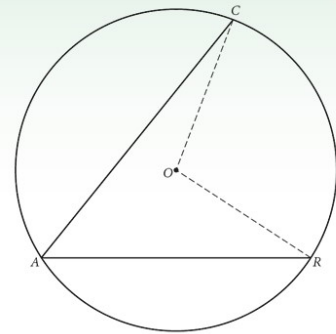




### INVESTIGATION 1 SOLUTION

**Step 1** Measure  $\angle COR$  with your protractor to find  $m\widehat{CR}$ , the intercepted arc. Measure  $\angle CAR$ . How does  $m\angle CAR$  compare with  $m\widehat{CR}$ ?

$$100^\circ; 50^\circ; m\angle CAR = \frac{1}{2}m\angle COR$$



**Step 2** Construct a circle of your own with an inscribed angle. Draw and measure the central angle that intercepts the same arc. What is the measure of the inscribed angle? How do the two measures compare?

**Step 3** Share your results with others near you. Copy and complete the conjecture.

#### Inscribed Angle Conjecture

C-81

The measure of an angle inscribed in a circle is **one-half the measure of the intercepted arc**.



### INVESTIGATION 2

#### Inscribed Angles Intercepting the Same Arc

**YOU WILL NEED:**  
compass,  
straightedge,  
protractor

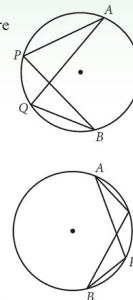
Next, let's consider two inscribed angles that intercept the same arc. In the figure at right,  $\angle AQB$  and  $\angle APB$  both intercept  $\widehat{AB}$ . Angles  $AQB$  and  $APB$  are both inscribed in  $\widehat{APB}$ .

**Step 1** Construct a large circle. Select two points on the circle. Label them  $A$  and  $B$ . Select a point  $P$  on the major arc and construct inscribed angle  $APB$ . With your protractor, measure  $\angle APB$ .

**Step 2** Select another point  $Q$  on  $\widehat{APB}$  and construct inscribed angle  $AQB$ . Measure  $\angle AQB$ .

**Step 3** How does  $m\angle AQB$  compare with  $m\angle APB$ ?

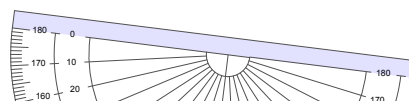
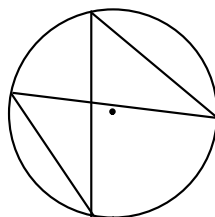
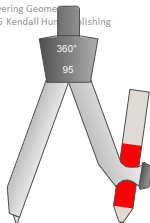
**Step 4** Repeat Steps 1–3 with points  $P$  and  $Q$  selected on minor arc  $AB$ . Compare results with your group. Then copy and complete the conjecture.



#### Inscribed Angles Intercepting Arcs Conjecture

C-82

Inscribed angles that intercept the same arc \_\_\_\_\_.





## INVESTIGATION 2

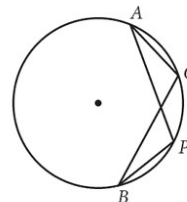
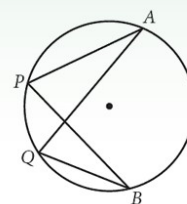
### SOLUTION

**Step 1** Construct a large circle. Select two points on the circle. Label them  $A$  and  $B$ . Select a point  $P$  on the major arc and construct inscribed angle  $APB$ . With your protractor, measure  $\angle APB$ .

**Step 2** Select another point  $Q$  on  $\widehat{APB}$  and construct inscribed angle  $AQB$ . Measure  $\angle AQB$ .

**Step 3** How does  $m\angle AQB$  compare with  $m\angle APB$ ? **equal**

**Step 4** Repeat Steps 1–3 with points  $P$  and  $Q$  selected on minor arc  $AB$ . Compare results with your group. Then copy and complete the conjecture.



#### Inscribed Angles Intercepting Arcs Conjecture

**C-82**

Inscribed angles that intercept the same arc **are congruent**.

## Homework Questions:

9.1 Tangent Properties Examples (Tue.)

Workbook p. 66 (Wed.)

## Practice: Midchapter Review

Note changes:

#1: skip length of  $\widehat{BC}$

#8: change numbers (make 25  $\rightarrow$  8 and 45  $\rightarrow$  16)

### Assignment:

Finish Midchapter Review

Textbook p. 520: 1-5

*Also make sure  
p 66 in workbook  
is complete*

**Reminder:** Friday, March 9

**Quiz** - 10.1/10.8; 9.1-9.3

