LESSON 9.3

Arcs and Angles

Objectives

- Discover relationships between an inscribed angle of the circle and its intercepted arc

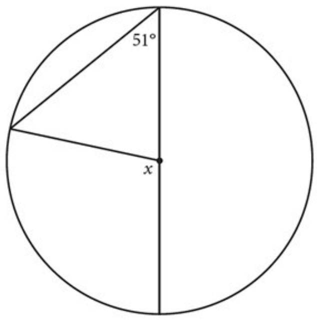
Please choose a compass and a protractor from the front.

LESSON 9.3

Arcs and Angles

Launch

Find x.



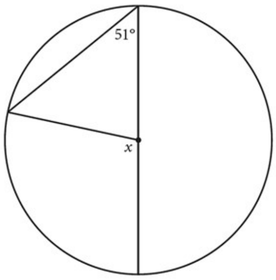
LESSON 9.3

Arcs and Angles

Launch

Find x.

$x = 102°$



LESSON 9.3

Arcs and Angles

Many arches that you see in structures are semicircular, but Chinese builders long ago discovered that arches don't have to have this shape. The Zhaozhou bridge, shown below, was completed in 605 c.e. It is the world's first stone arched bridge in the shape of a minor arc, predating other minor-arc arches by about 800 years.

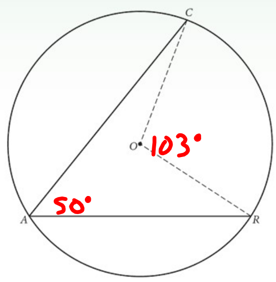
In this lesson you'll discover properties of arcs and the angles associated with them.

INVESTIGATION 1

Inscribed Angle Properties

YOU WILL NEED:
compass,
straightedge,
protractor

In this investigation you will compare an inscribed angle and a central angle, both inscribed in the same arc. Refer to the diagram of circle *O*, with central angle *COR* and inscribed angle *CAR*.



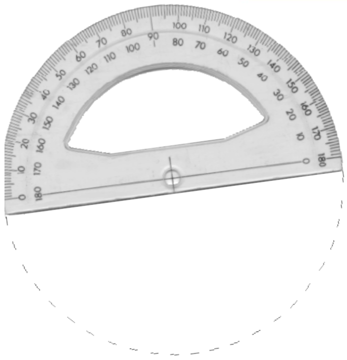
Step 1 Measure $\angle COR$ with your protractor to find $m\widehat{CR}$, the intercepted arc. Measure $\angle CAR$. How does $m\angle CAR$ compare with $m\widehat{CR}$?

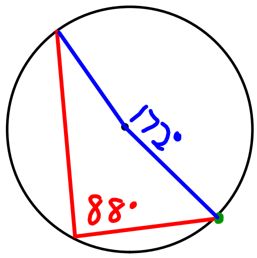
Step 2 Construct a circle of your own with an inscribed angle. Draw and measure the central angle that intercepts the same arc. What is the measure of the inscribed angle? How do the two measures compare?

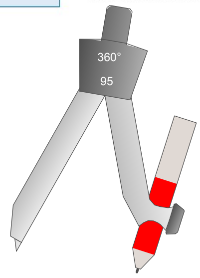
Step 3 Share your results with others near you. Copy and complete the conjecture.

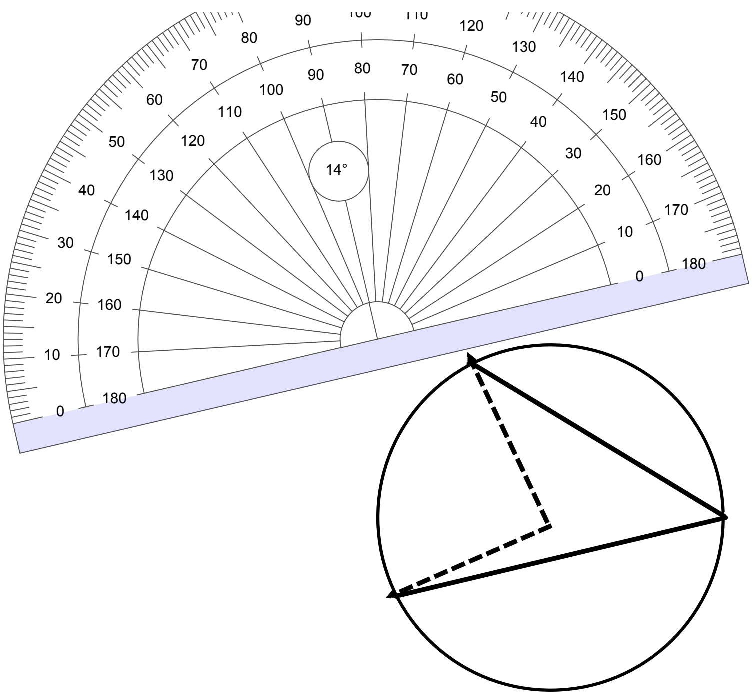
Inscribed Angle Conjecture **C-81**

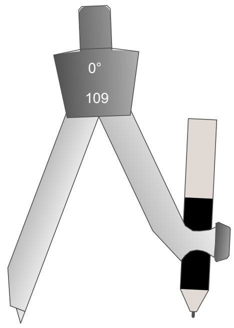
The measure of an angle inscribed in a circle _____.





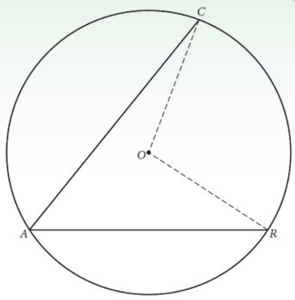






INVESTIGATION 1
SOLUTION

Step 1 Measure $\angle COR$ with your protractor to find $m\widehat{CR}$, the intercepted arc. Measure $\angle CAR$. How does $m\angle CAR$ compare with $m\widehat{CR}$?

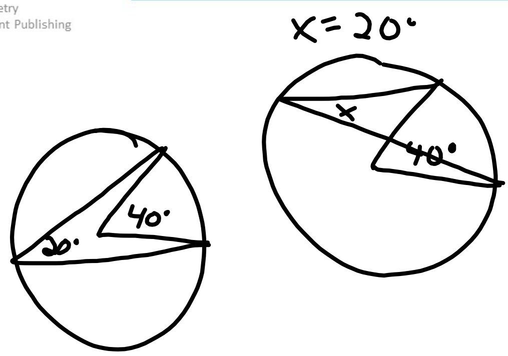


100°; 50°; $m\angle CAR = \frac{1}{2} m\angle COR$

Step 2 Construct a circle of your own with an inscribed angle. Draw and measure the central angle that intercepts the same arc. What is the measure of the inscribed angle? How do the two measures compare?

Step 3 Share your results with others near you. Copy and complete the conjecture.

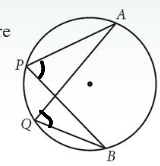
> **Inscribed Angle Conjecture** **C-81**
>
> The measure of an angle inscribed in a circle is one-half the measure of the intercepted arc.



INVESTIGATION 2
Inscribed Angles Intercepting the Same Arc

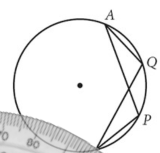
YOU WILL NEED: compass, straightedge, protractor

Next, let's consider two inscribed angles that intercept the same arc. In the figure at right, $\angle AQB$ and $\angle APB$ both intercept $\widehat{AB}$. Angles AQB and APB are both inscribed in $\widehat{APB}$.



Step 1 Construct a large circle. Select two points on the circle. Label them A and B. Select a point P on the major arc and construct inscribed angle APB. With your protractor, measure $\angle APB$.

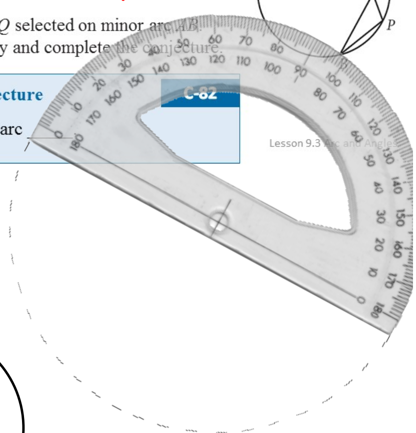
Step 2 Select another point Q on $\widehat{APB}$ and construct inscribed angle AQB. Measure $\angle AQB$.

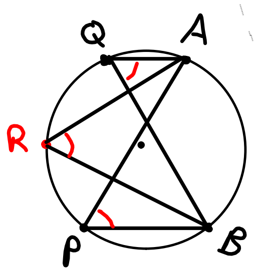


Step 3 How does $m\angle AQB$ compare with $m\angle APB$? They are ≅

Step 4 Repeat Steps 1–3 with points P and Q selected on minor arc AB. Compare results with your group. Then copy and complete the conjecture.

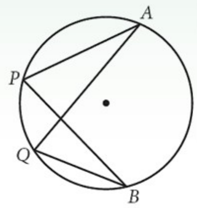
> **Inscribed Angles Intercepting Arcs Conjecture** **C-82**
>
> Inscribed angles that intercept the same arc





INVESTIGATION 2
SOLUTION

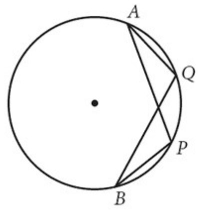
Step 1 Construct a large circle. Select two points on the circle. Label them *A* and *B*. Select a point *P* on the major arc and construct inscribed angle *APB*. With your protractor, measure $\angle APB$.



Step 2 Select another point *Q* on $\overset{\frown}{APB}$ and construct inscribed angle *AQB*. Measure $\angle AQB$.

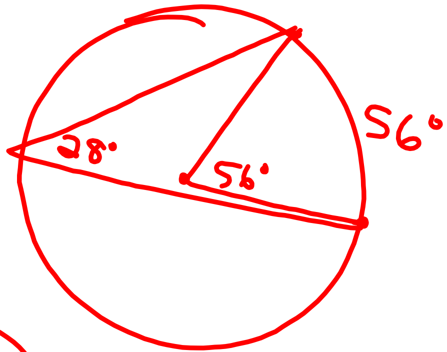
Step 3 How does $m\angle AQB$ compare with $m\angle APB$? equal

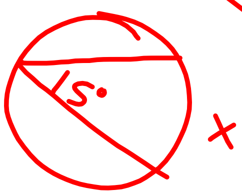
Step 4 Repeat Steps 1–3 with points *P* and *Q* selected on minor arc *AB*. Compare results with your group. Then copy and complete the conjecture.



Inscribed Angles Intercepting Arcs Conjecture **C-82**

Inscribed angles that intercept the same arc are congruent.

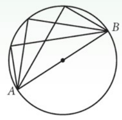




INVESTIGATION 3
Angles Inscribed in a Semicircle

YOU WILL NEED:
compass,
straightedge,
protractor

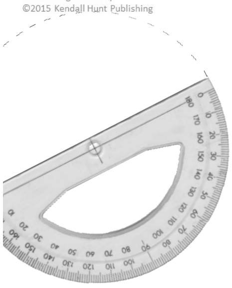
Next, you will investigate a property of angles inscribed in semicircles. This will lead you to a third important conjecture about inscribed angles.

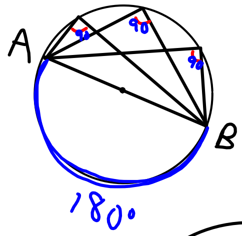


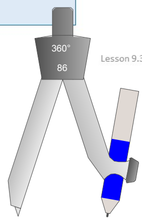
Step 1 Construct a large circle. Construct a diameter $\overline{AB}$. Inscribe three angles in the same semicircle. Make sure the sides of each angle pass through A and B.

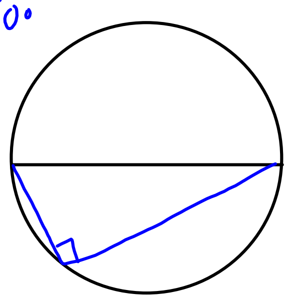
Step 2 Measure each angle with your protractor. What do you notice? Compare your results with the results of others and make a conjecture.

Angles Inscribed in a Semicircle Conjecture	C-83
Angles inscribed in a semicircle ______.	



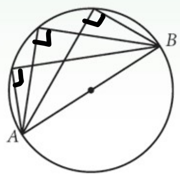






INVESTIGATION 3
SOLUTION

Step 1 Construct a large circle. Construct a diameter $\overline{AB}$. Inscribe three angles in the same semicircle. Make sure the sides of each angle pass through A and B.



Step 2 Measure each angle with your protractor. What do you notice? Compare your results with the results of others and make a conjecture.

Angles Inscribed in a Semicircle Conjecture	C-83
Angles inscribed in a semicircle are right angles.	

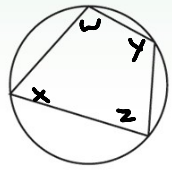
Now you will discover a property of the angles of a quadrilateral inscribed in a circle.

INVESTIGATION 4

Cyclic Quadrilaterals

YOU WILL NEED:
compass,
straightedge,
protractor

A quadrilateral inscribed in a circle is called a **cyclic quadrilateral**. Each of its angles is inscribed in the circle, and each of its sides is a chord of the circle.



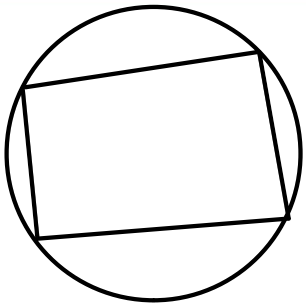
$x+y=180°$

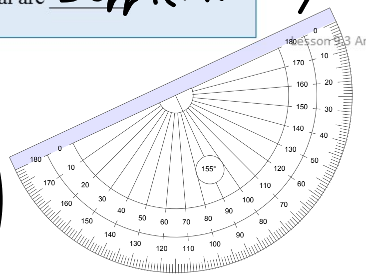
$w+z=180°$

Step 1 Construct a large circle. Construct a cyclic quadrilateral by connecting four points anywhere on the circle.

Step 2 Measure each of the four inscribed angles. Write the measure in each angle. Look carefully at the sums of various angles. Share your observations with students near you. Then copy and complete the conjecture.

> **Cyclic Quadrilateral Conjecture** **C-84**
>
> The opposite angles of a cyclic quadrilateral are Supplementary

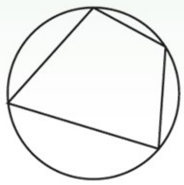




INVESTIGATION 4

SOLUTION

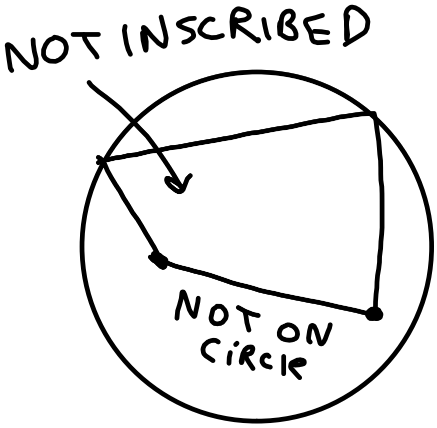
A quadrilateral inscribed in a circle is called a **cyclic quadrilateral**. Each of its angles is inscribed in the circle, and each of its sides is a chord of the circle.



Step 1 Construct a large circle. Construct a cyclic quadrilateral by connecting four points anywhere on the circle.

Step 2 Measure each of the four inscribed angles. Write the measure in each angle. Look carefully at the sums of various angles. Share your observations with students near you. Then copy and complete the conjecture.

> **Cyclic Quadrilateral Conjecture** **C-84**
>
> The opposite angles of a cyclic quadrilateral are supplementary.



LESSON 9.3

Arcs and Angles

Review these conjectures and ask yourself which quadrilaterals can be inscribed in a circle. Can any parallelogram be a cyclic quadrilateral? If two sides of a cyclic quadrilateral are parallel, then what kind of quadrilateral will it be?

LESSON 9.3

Arcs and Angles

Summarize

LESSON 9.3

Arcs and Angles

Summarize

- How can the Inscribed Angle Conjecture be used to explain *why* the other conjectures are true?
- If the inscribed angle is obtuse, what is the measure of the intercepted arc?
- Which quadrilaterals are cyclic?
- Can a parallelogram be a cyclic quadrilateral?
- If two sides of a cyclic quadrilateral are parallel, what kind of quadrilateral can it be?

LESSON 9.3

Arcs and Angles

Summarize

- How can the Inscribed Angle Conjecture be used to explain *why* the other conjectures are true?

An angle inscribed in a semicircle intercepts a 180° arc, so its measure will be 90°. Opposite angles of a cyclic quadrilateral intercept arcs whose measures sum to 360°, so the sum of the angle measures will be half that, or 180°. And if a diagonal is drawn across the quadrilateral-like shape formed by two parallel lines and their intercepted arcs, the two inscribed angles will be congruent by the Parallel Lines Conjecture, so the intercepted arcs will be congruent.

- If the inscribed angle is obtuse, what is the measure of the intercepted arc?

More than 180°.

LESSON 9.3

Arcs and Angles

Summarize

- Which quadrilaterals are cyclic?

All squares and rectangles and some kites, trapezoids, and nonspecial quadrilaterals.

- Can a parallelogram be a cyclic quadrilateral?

Only if it's a rectangle.

- If two sides of a cyclic quadrilateral are parallel, what kind of quadrilateral can it be?

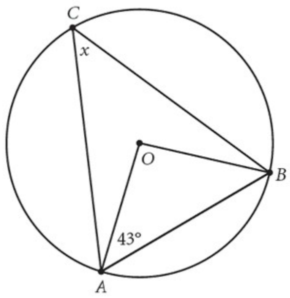
Rectangle, square, or isosceles trapezoid.

LESSON
9.3

Arcs and Angles

Closing Question

Given Circle O. Find x.



LESSON
9.3

Arcs and Angles

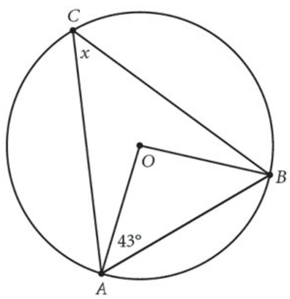
Closing Question

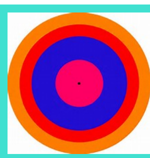
ANSWER

Given Circle O. Find x.

$x = 47^{\circ}$

No Homework

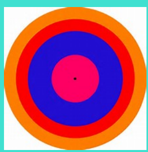




Day 2:

Please take out your graph paper with the four completed constructions from yesterday.

You will be working with your group to complete the problems in the packet.

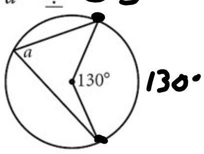


9.3 Exercises

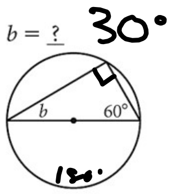
pages 468 – 470

Use your new conjectures to solve Exercises 1–17. For each exercise, explain how you determined your answer.

1. $a = \underline{?}$ 65°

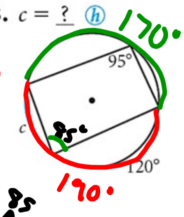


2. $b = \underline{?}$ 30°



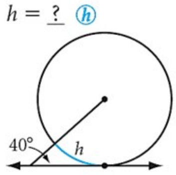
3. $c = \underline{?}$ (h)

C+120=190
C=70

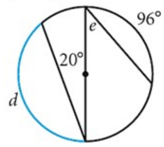


85
180-95

4. $h = \underline{?}$ (h)



5. $d = \underline{?}$
$e = \underline{?}$



9.3 Exercises
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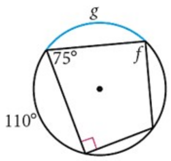
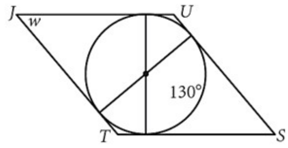
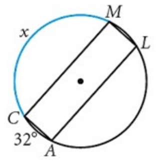
6. $f = \underline{?}$
$g = \underline{?}$



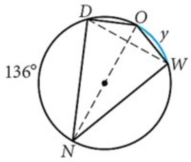
7. *JUST* is a rhombus.
$w = \underline{?}$



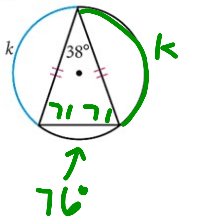
8. *CALM* is a rectangle.
$x = \underline{?}$



9. *DOWN* is a kite.
$y = \underline{?}$

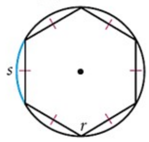


10. $k = \underline{?}$



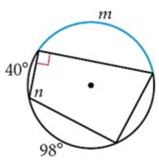
$2k+76=360$
$2k=284$
$k=142$

11. $r = \underline{?}$
$s = \underline{?}$

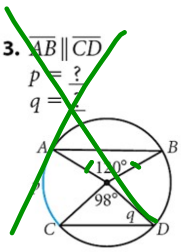


9.3 Exercises
pages 468 – 470

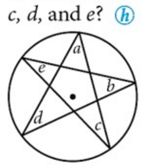
12. $m = \underline{?}$
$n = \underline{?}$



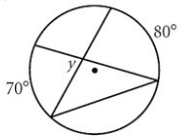
13. $\overline{AB} \parallel \overline{CD}$
$p = \underline{?}$
$q = \underline{?}$



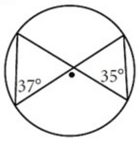
14. What is the sum of *a*, *b*, *c*, *d*, and *e*? (h)



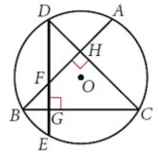
15. $y = \underline{?}$ (h)



16. What's wrong with this picture?



17. Is $\widehat{AC} \cong \widehat{CE}$? Explain.



9.3 Exercises

pages 468 – 470

18. What is the difference between "an angle inscribed in an arc" and "an angle that intercepts an arc"? Draw and label an example of each.

19. How can you find the center of a circle, using only the corner of a piece of paper?

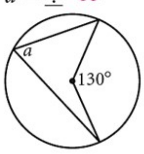
20. Chris Chisholm, a high school student in Whitmore, California, used the Angles Inscribed in a Semicircle Conjecture to discover a simpler way to find the orthocenter in a triangle. Chris constructs a circle using one of the sides of the triangle as the diameter, then immediately finds an altitude to each of the triangle's other two sides. Use geometry software and Chris's method to find the orthocenter of a triangle. Does this method work on all kinds of triangles? ⓗ

ANSWERS

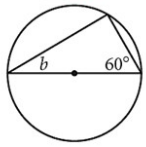
9.3 Exercises

Use your new conjectures to solve Exercises 1–17. For each exercise, explain how you determined your answer.

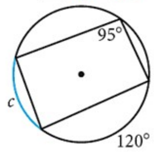
1. $a =$? 65°



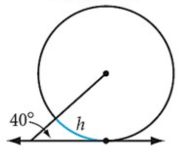
2. $b =$? 30°



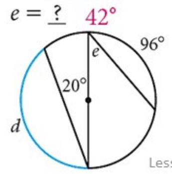
3. $c =$? ⓗ 70°



4. $h =$? ⓗ 50°



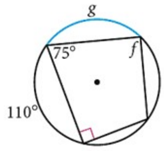
5. $d =$? 140°
$e =$? 42°



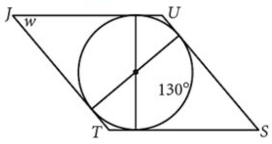
ANSWERS

9.3 Exercises

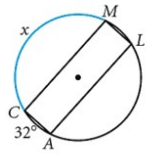
6. $f = \underline{?}$ 90°
$g = \underline{?}$ 100°



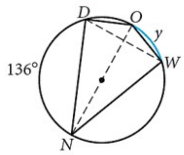
7. *JUST* is a rhombus.
$w = \underline{?}$ 50°



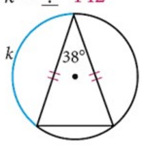
8. *CALM* is a rectangle.
$x = \underline{?}$ 148°



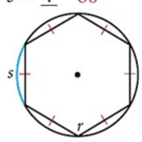
9. *DOWN* is a kite.
$y = \underline{?}$ 44°



10. $k = \underline{?}$ 142°



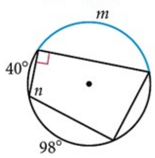
11. $r = \underline{?}$ 120°
$s = \underline{?}$ 60°



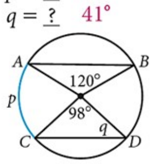
ANSWERS

9.3 Exercises

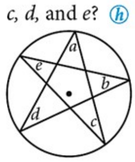
12. $m = \underline{?}$ 140°
$n = \underline{?}$ 111°



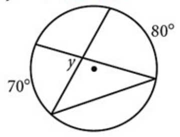
13. $\overline{AB} \parallel \overline{CD}$
$p = \underline{?}$ 71°
$q = \underline{?}$ 41°



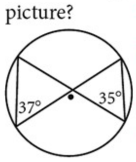
14. What is the sum of *a*, *b*, *c*, *d*, and *e*? ⓗ 180°



15. $y = \underline{?}$ ⓗ 75°

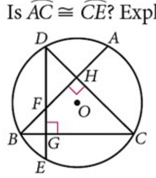


16. What's wrong with this picture?



The two inscribed angles intercept the same arc, so they should be congruent.

17. Is $\widehat{AC} \cong \widehat{CE}$? Explain.



$\angle BFE \cong \angle DFA$
(Vertical Angles Conjecture).
$\angle BGD \cong \angle FHD$
(all right angles congruent).
Therefore, $\angle B \cong \angle D$
(Third Angle Conjecture).
$m\angle B = \frac{1}{2} m\widehat{AC}$
$m\angle D = \frac{1}{2} m\widehat{EC}$
$\widehat{AC} \cong \widehat{EC}$

ANSWERS

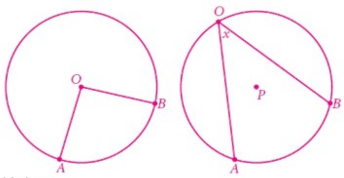
9.3 Exercises

18. What is the difference between "an angle inscribed in an arc" and "an angle that intercepts an arc"? Draw and label an example of each.

19. How can you find the center of a circle, using only the corner of a piece of paper?

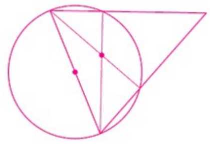
20. Chris Chisholm, a high school student in Whitmore, California, used the Angles Inscribed in a Semicircle Conjecture to discover a simpler way to find the orthocenter in a triangle. Chris constructs a circle using one of the sides of the triangle as the diameter, then immediately finds an altitude to each of the triangle's other two sides. Use geometry software and Chris's method to find the orthocenter of a triangle. Does this method work on all kinds of triangles? (h)

18. An inscribed angle is an angle with its vertex on the circle, formed by intersecting chords. An angle that intercepts an arc could be an inscribed angle formed by two chords, an angle formed by a tangent and a chord, a central angle formed by two radii, angles forms by intersecting chords, or angles formed by tangents and secants. Possible examples: Circle *O* has central angle *AOB* which intercepts arc *AB*. Circle *P* has inscribed angle *AOB* which intercepts arc *AB*.



19. Possible answer: Place the corner so that it is an inscribed angle. Trace the inscribed angle. Use the side of the paper to construct the hypotenuse of the right triangle (which is the diameter). Repeat the process. The place where the two diameters intersect is the center.

20. possible answer:



It works on acute and right triangles.

Homework:

Workbook page 67