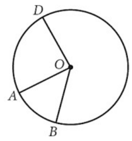
INVESTIGATION 1

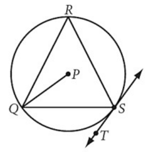
Defining Angles in a Circle

Write a good definition of each boldfaced term. Discuss your definitions with others in your group. Agree on a common set of definitions as a class and add them to your definition list. In your notebook, draw and label a figure to illustrate each term.

Step 1 **Central Angle**

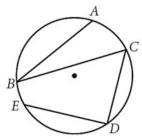


∠*AOB*, ∠*DOA*, and ∠*DOB* are central angles of circle *O*.

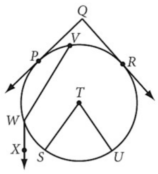


∠*PQR*, ∠*PQS*, ∠*RST*, ∠*QST*, and ∠*QSR* are not central angles of circle *P*

Step 2 **Inscribed Angle**



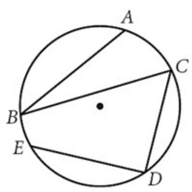
∠*ABC*, ∠*BCD*, and ∠*CDE* are inscribed angles.



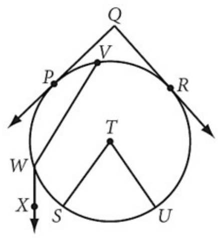
∠*PQR*, ∠*STU*, and ∠*VWX* are not inscribed angles.

Launch

Step 2 **Inscribed Angle**



∠*ABC*, ∠*BCD*, and ∠*CDE* are inscribed angles.

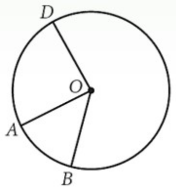


∠*PQR*, ∠*STU*, and ∠*VWX* are not inscribed angles.

INVESTIGATION 1
SOLUTION

Launch

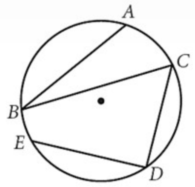
Step 1 Central Angle



$\angle AOB$, $\angle DOA$, and $\angle DOB$ are central angles of circle O.

A central angle has its vertex at the center of the circle.

Step 2 Inscribed Angle



$\angle ABC$, $\angle BCD$, and $\angle CDE$ are inscribed angles.

An inscribed angle has its vertex on the circle and its sides are chords.

LESSON 9.2

Chord Properties

COMMON CORE STATE STANDARDS		
Applied	**Developed**	**Introduced**
	G.C.2	

G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

LESSON 9.2

Chord Properties

Objectives

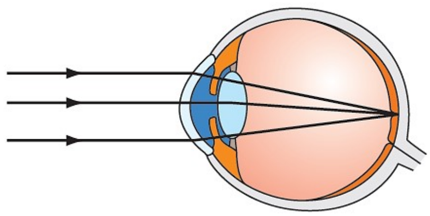
- Discover properties of chords to a circle

LESSON 9.2

Chord Properties

In the last lesson you discovered some properties of a tangent, a line that intersects the circle only once. In this lesson you will investigate properties of a chord, a line segment whose endpoints lie on the circle.

In a person with correct vision, light rays from distant objects are focused to a point on the retina. If the eye represents a circle, then the path of the light from the lens to the retina represents a chord. The angle formed by two of these chords to the same point on the retina represents an inscribed angle. How would you define an inscribed angle?



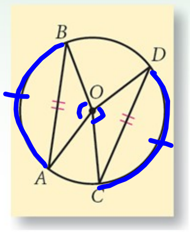
Before investigating the properties of chords in a circle, let's define two types of angles in a circle.

INVESTIGATION 2

Chords and Their Central Angles

YOU WILL NEED: compass, straightedge, protractor, patty paper (optional)

Next you will discover some properties of chords and central angles. You will also see a relationship between chords and arcs.



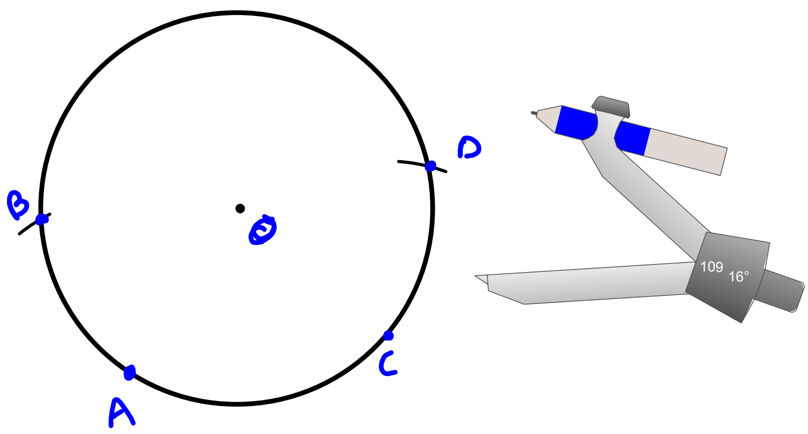
Step 1 Construct a large circle. Label the center O. Construct two congruent chords in your circle. Label the chords $\overline{AB}$ and $\overline{CD}$, then construct radii $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, and $\overline{OD}$.

Step 2 With your protractor, measure $\angle BOA$ and $\angle COD$. How do they compare? Share your results with others in your group. Then copy and complete the conjecture.

Chord Central Angles Conjecture **C-76**

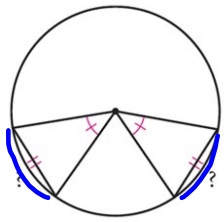
If two chords in a circle are congruent, then they determine two central angles that are ______.

congruent



Step 3 How can you fold your circle construction to check the conjecture?

Step 4 Recall that the measure of an arc is defined as the measure of its central angle. If two central angles are congruent, their intercepted arcs must be congruent. Combine this fact with the Chord Central Angles Conjecture to complete the next conjecture.



Chord Arcs Conjecture **C-77**

If two chords in a circle are congruent, then their ______ are congruent.

intercepted arcs

INVESTIGATION 2
SOLUTION

Chord Central Angles Conjecture **C-76**

If two chords in a circle are congruent, then they determine two central angles that are congruent.

Step 3 How can you fold your circle construction to check the conjecture?

Fold it so that $\overline{OD}$ coincides with $\overline{OB}$ and $\overline{OC}$ coincides with $\overline{OA}$.

Chord Arcs Conjecture **C-77**

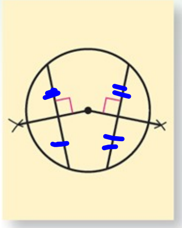
If two chords in a circle are congruent, then their intercepted arcs are congruent.

INVESTIGATION 3

Chords and the Center of the Circle

YOU WILL NEED: compass, straightedge, patty paper (optional)

In this investigation you will discover relationships about a chord and the center of its circle.

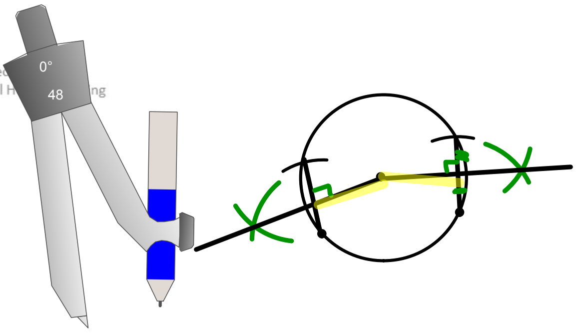


Step 1 Construct a large circle and mark the center. Construct two nonparallel congruent chords. Then construct the perpendiculars from the center to each chord.

Step 2 How does the perpendicular from the center of a circle to a chord divide the chord? Copy and complete the conjecture.

Perpendicular to a Chord Conjecture **C-78**

The perpendicular from the center of a circle to a chord is the bisector of the chord.



INVESTIGATION 3

Chords and the Center of the Circle

YOU WILL NEED: compass, straightedge, patty paper (optional)

Let's continue this investigation to discover a relationship between the length of congruent chords and their distances from the center of the circle.

Step 3 Compare the distances (measured along the perpendicular) from the center to the chords. Are the results the same if you change the size of the circle and the length of the chords? State your observations as your next conjecture.

Chord Distance to Center Conjecture **C-79**

Two congruent chords in a circle are equidistant from the center of the circle.

INVESTIGATION 3
SOLUTION

Perpendicular to a Chord Conjecture **C-78**

The perpendicular from the center of a circle to a chord is the bisector of the chord.

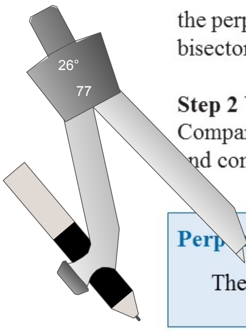
Chord Distance to Center Conjecture **C-79**

Two congruent chords in a circle are equidistant from the center of the circle.

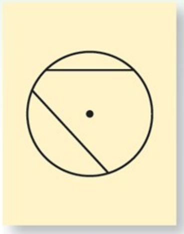
INVESTIGATION 4

Perpendicular Bisector of a Chord

YOU WILL NEED:
compass,
straightedge,
patty paper
(optional)



Next, you will discover a property of perpendicular bisectors of chords.

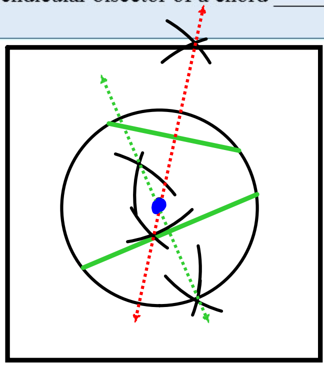


Step 1 Construct a large circle and mark the center. Construct two nonparallel chords that are not diameters. Then construct the perpendicular bisector of each chord and extend the bisectors until they intersect.

Step 2 What do you notice about the point of intersection? Compare your results with the results of others near you. Copy and complete the conjecture.

Perpendicular Bisector of a Chord Conjecture **C-80**

The perpendicular bisector of a chord __________.



INVESTIGATION 4
SOLUTION

Perpendicular Bisector of a Chord Conjecture **C-80**

The perpendicular bisector of a chord passes through the center of the circle.

With the perpendicular bisector of a chord, you can find the center of any circle, and therefore the vertex of the central angle to any arc. All you have to do is construct the perpendicular bisectors of nonparallel chords.

"Pull the cord?! Don't I need to construct it first?"

LESSON 9.2

Chord Properties

Summarize

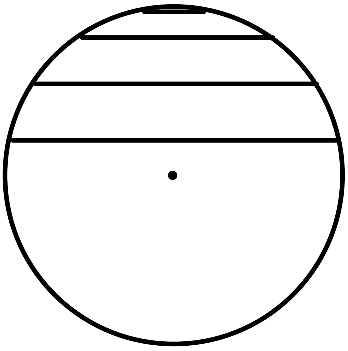
LESSON 9.2

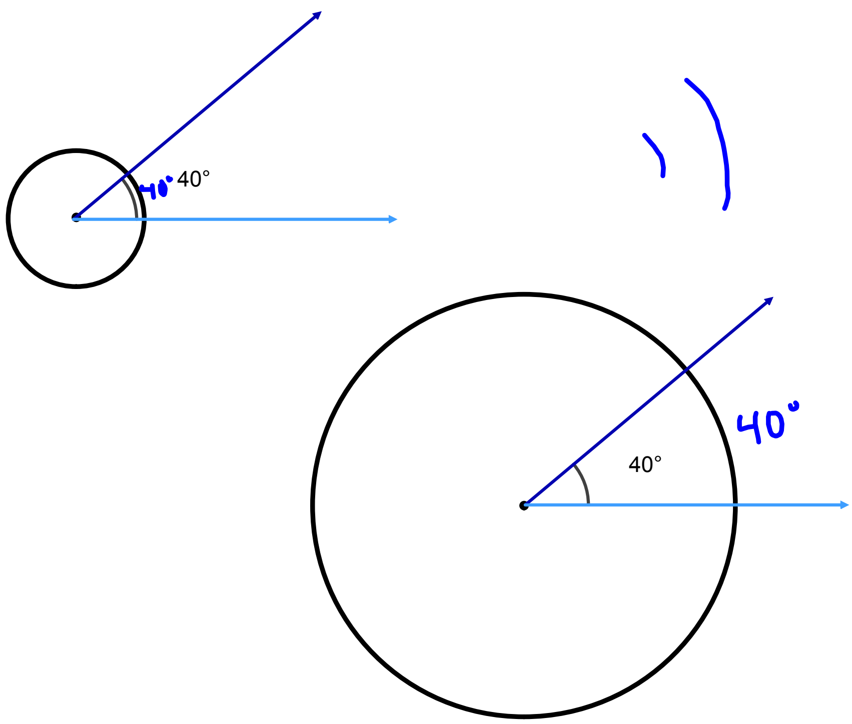
Chord Properties

Summarize

- Congruent chords are equidistant from the center. Can we say anything about distance from the center if one chord is longer than the other?

- What's needed to ensure that two arcs have the same size and shape?





LESSON 9.2

Chord Properties

Summarize

- Congruent chords are equidistant from the center. Can we say anything about distance from the center if one chord is longer than the other?

  In the same circle, shorter chords are farther from the center.

- What's needed to ensure that two arcs have the same size and shape?

  Congruent arcs must be on the same or congruent circles.

9.2

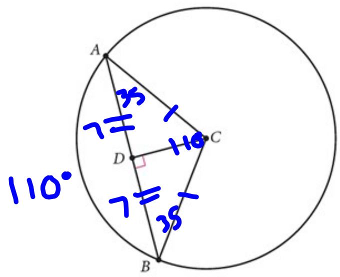
Chord Properties

180-110=70

Extra Example

$AB = 14$ cm $\quad \widehat{AB} = 110°$

$\angle CAD = ?$ $\quad DB = ?$



LESSON 9.2

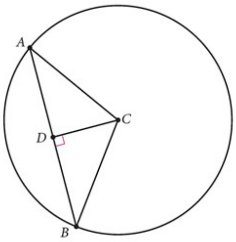
Chord Properties

Extra Example

ANSWER

$AB = 14$ cm $\widehat{AB} = 110°$
$\angle CAD = ?$ $DB = ?$

$\angle CAD = 35°$ $DB = 7$ cm

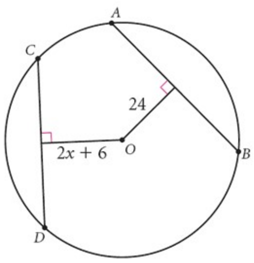


LESSON 9.2

Chord Properties

Closing Question

Given Circle P with $AB \cong CD$.
Find x.



LESSON 9.2

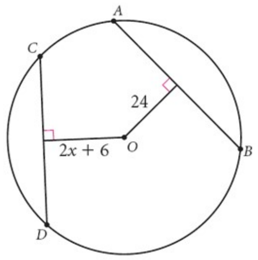
Chord Properties

Closing Question

ANSWER

Given Circle P with $AB \cong CD$.
Find x.

$x = 9$



Homework: Workbook Pg. 66