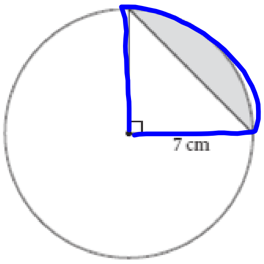
Warm-Up: Find the exact area of the segment below.

Area of shaded segment = _____



$12.25\pi - 24.5 \text{ cm}^2$

$\frac{90}{360}\pi \cdot 7^2 \quad \frac{1}{2} \cdot 7 \cdot 7$

LESSON 8.4 • Areas of Sectors

1. $\frac{25\pi}{12}$ cm^2 ≈ 6.54 cm^2
2. $\frac{32\pi}{3}$ cm^2 ≈ 33.51 cm^2
3. 12π cm^2 ≈ 37.70 cm^2
4. $(16\pi - 32)$ cm^2 ≈ 18.27 cm^2
5. 13.5π cm^2 ≈ 42.41 cm^2
6. 10π cm^2 ≈ 31.42 cm^2
7. $r = 10$ cm
8. $x = 135°$
9. $r = 7$ cm

LESSON 8.6

Area and Similarity

Objectives

- Discover the relationship between the areas of similar figures
- Discover the relationship between the surface areas of similar solids
- Apply the similarity conjectures to problems involving area

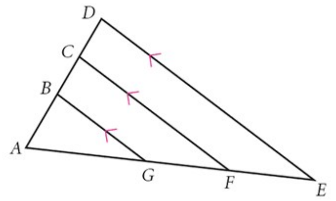
LESSON 8.6

Area and Similarity

Launch

Which triangles are similar?

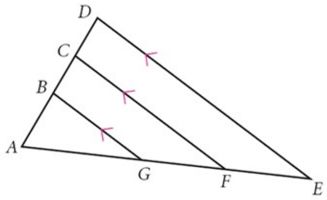
Why?



LESSON 8.6

Area and Similarity

Launch



Which triangles are similar?

$\Delta ADE \sim \Delta ACF \sim \Delta ABG$

Why?

AA Similarity Conjecture

INVESTIGATION 1

YOU WILL NEED:
graph paper

Area Ratios

In this investigation you will find the relationship between areas of similar figures. Have each member of your group pick a different whole number scale factor.

Step 1 Draw a rectangle on graph paper. Calculate its area.

Step 2 Multiply each dimension of your rectangle by your scale factor and draw the enlarged similar rectangle. Calculate its area.

Step 3 What is the ratio of side lengths (larger to smaller) for your two rectangles? What is the ratio of their areas (larger to smaller)?

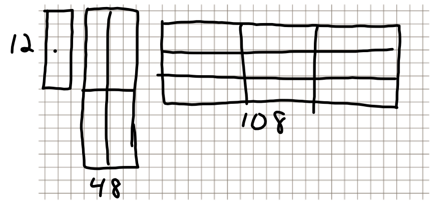
Step 4 How many copies of the smaller rectangle would you need to fill the larger rectangle? Draw lines in your larger rectangle to show how you would place the copies to fill the area.

Investigation 1 • Area and Similarity

Name ______________________ Period ________ Date ____________

In this investigation you will find the relationship between areas of similar figures. Have each member of your group pick a different whole number scale factor.

Step 1 Draw a small rectangle on the grid. Calculate its area.



Step 2 Multiply each dimension of your rectangle by your scale factor and draw the enlarged similar rectangle. Calculate its area.

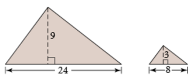
Sides $\frac{2}{1}$ Area $\frac{4}{1}$ Sides $\frac{3}{1}$ Area $\frac{108}{12} = \frac{9}{1}$

Step 3 What is the ratio of side lengths (larger to smaller) for your two rectangles? What is the ratio of their areas (larger to smaller)?

Step 4 How many copies of the smaller rectangle would you need to fill the larger rectangle? Draw lines in your larger rectangle to show how you would place the copies to fill the area.

Investigation 1 • Area and Similarity (continued)

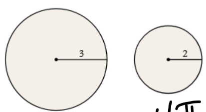
Step 5 Find the areas of these two similar triangles. Write the ratio of the smaller area to the larger area as a fraction and reduce. Do the same with the side lengths. Compare these two ratios.



A = 108 A = 12

$\frac{12}{108} = \frac{1}{9}$ Areas $\frac{8}{24} = \frac{1}{3}$ Sides

Step 6 Find the exact areas of these two circles in terms of π, not as approximate decimals. Write the ratio of the smaller area to the larger area as a fraction and reduce. Do the same with the radii. Compare these two ratios.



9π 4π

$\frac{4\pi}{9\pi} = \frac{4}{9}$ $\frac{2}{3}$

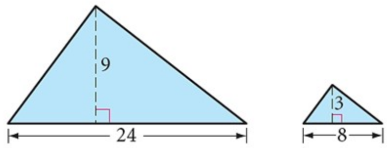
Step 7 Compare your results with those of others in your group and look for a pattern. This relationship between the ratio of corresponding sides (or radii) and the ratio of areas can be generalized to all similar polygons because all polygons can be divided into triangles. You should be ready to state a conjecture.

Proportional Areas Conjecture

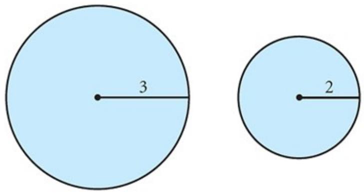
If corresponding side lengths of two similar polygons or the radii of two circles compare in the ratio $\frac{m}{n}$, then their areas compare in the ratio ______________.

$\frac{m^2}{n^2}$ or $\left(\frac{m}{n}\right)^2$

Step 5 Find the areas of these two similar triangles. Write the ratio of the smaller area to the larger area as a fraction and simplify. Do the same with the side lengths. Compare these two ratios.



Step 6 Find the exact areas of these two circles in terms of π, not as approximate decimals. Write the ratio of the smaller area to the larger area as a fraction and simplify. Do the same with the radii. Compare these two ratios.



Step 7 Compare your results with those of others in your group and look for a pattern. This relationship between the ratio of corresponding sides (or radii) and the ratio of areas can be generalized to all similar polygons because all polygons can be divided into triangles. You should be ready to state a conjecture.

Proportional Areas Conjecture **C-73**

If corresponding side lengths of two similar polygons or the radii of two circles compare in the ratio $\frac{m}{n}$, then their areas compare in the ratio ______.

INVESTIGATION 1
SOLUTION

Step 1 Draw a rectangle on graph paper. Calculate its area.

Possible answer: A 3-by-5 rectangle would have an area of 15.

Step 2 Multiply each dimension of your rectangle by your scale factor and draw the enlarged similar rectangle. Calculate its area.

Possible answer: Using a scale factor of 2, the new rectangle would be 6 by 10. The area would be 60.

Step 3 What is the ratio of side lengths (larger to smaller) for your two rectangles? What is the ratio of their areas (larger to smaller)?

Possible answer: Ratio of side lengths is $\frac{2}{1}$. Ratio of areas is $\frac{4}{1}$, or $\frac{2^2}{1^2}$.

Step 4 How many copies of the smaller rectangle would you need to fill the larger rectangle? Draw lines in your larger rectangle to show how you would place the copies to fill the area.

Possible answer: You would need four copies.

Step 5 Find the areas of these two similar triangles. Write the ratio of the smaller area to the larger area as a fraction and simplify. Do the same with the side lengths. Compare these two ratios.

area of large triangle = 108 square units; area of small triangle = 12 square units; area ratio = $\frac{1}{9}$; side length ratio = $\frac{1}{3}$

Step 6 Find the exact areas of these two circles in terms of π, not as approximate decimals. Write the ratio of the smaller area to the larger area as a fraction and simplify. Do the same with the radii. Compare these two ratios.

area of large circle = 9π square units; area of small circle = 4π square units; area ratio = $\frac{4}{9}$; ratio of radii = $\frac{2}{3}$

Step 7 Compare your results with those of others in your group and look for a pattern. This relationship between the ratio of corresponding sides (or radii) and the ratio of areas can be generalized to all similar polygons because all polygons can be divided into triangles. You should be ready to state a conjecture.

Proportional Areas Conjecture **C-73**

If corresponding side lengths of two similar polygons or the radii of two circles compare in the ratio $\frac{m}{n}$, then their areas compare in the ratio $\frac{m^2}{n^2}$ or $\left(\frac{m}{n}\right)^2$.

EXAMPLE

If you need 3 oz of shredded cheese to cover a medium 12 in. diameter pizza, how much shredded cheese would you need to cover a large 16 in. diameter pizza?

Ratio of diameters = $\frac{12}{16} = \frac{3}{4}$

Ratio of areas = $\frac{9}{16}$

$\frac{3}{x} = \frac{9}{16}$

$9x = 48$

$x = \frac{48}{9} = 5.\overline{3} = \boxed{5\frac{1}{3}\text{ oz}}$

SOLUTION

The ratio of the pizza diameters is $\frac{16}{12}$, or $\frac{4}{3}$. Therefore the ratio of the pizza areas is $\left(\frac{4}{3}\right)^2$, or $\frac{16}{9}$. Let x represent the amount of cheese needed for a large pizza and set up a proportion.

$$\frac{\text{area of large pizza}}{\text{area of small pizza}} = \frac{\text{amount of cheese for large pizza}}{\text{amount of cheese for small pizza}}$$

$$\frac{16}{9} = \frac{x}{3}$$

By solving for x, you find that a large pizza requires $5\frac{1}{3}$ oz of shredded cheese.

Example: A model of a building has a scale factor of 1:20 with the actual building. What is the ratio of the area of the model's roof to the area of the actual roof of the building?

Scale factor^2 = ratio areas

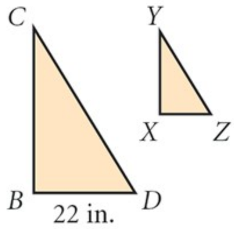
1:400

Extra Example

$\Delta BCD \sim \Delta XYZ$

$$\frac{\text{area of } \Delta BCD}{\text{area of } \Delta XYZ} = \frac{4}{1}$$

$XZ = ?$



$$\left(\frac{22}{x}\right)^2 = \frac{4}{1}$$

$$\frac{484}{x^2} = \frac{4}{1}$$

$4x^2 = 484$

$x^2 = 121$

$x = 11$

Ratio of sides $= \frac{2}{1}$

$$\frac{22}{x} = \frac{2}{1}$$

$2x = 22$

$x = 11$ in

LESSON 8.6

Area and Similarity

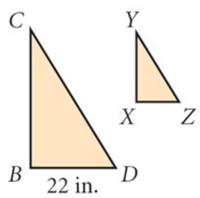
Extra Example

ANSWER

$\Delta BCD \sim \Delta XYZ$

$$\frac{\text{area of } \Delta BCD}{\text{area of } \Delta XYZ} = \frac{4}{1}$$

$XZ = 11$ in.



LESSON 8.6

Area and Similarity

Closing Question

The ratio of the areas of two similar trapezoids is 1:9.

What is the ratio of the lengths of their altitudes?

LESSON 8.6

Area and Similarity

Closing Question

ANSWER

The ratio of the areas of two similar trapezoids is 1:9.

What is the ratio of the lengths of their altitudes? 1 : 3

LESSON 8.6

Area and Similarity

Summarize

- If two figures are similar, what is the ratio of any corresponding two-dimensional parts?
- All circles are similar to one another, as dilations of one another. Are there polygons that are similar to all others of the same name?

LESSON 8.6

Area and Similarity

Summarize

- If two figures are similar, what is the ratio of any corresponding two-dimensional parts?

  The ratio is the square of the scale factor.

- All circles are similar to one another, as dilations of one another. Are there polygons that are similar to all others of the same name?

  Squares, regular hexagons, equilateral triangles, isosceles right triangles, and so on.

Homework: Pg. 64 1-9 (Workbook)