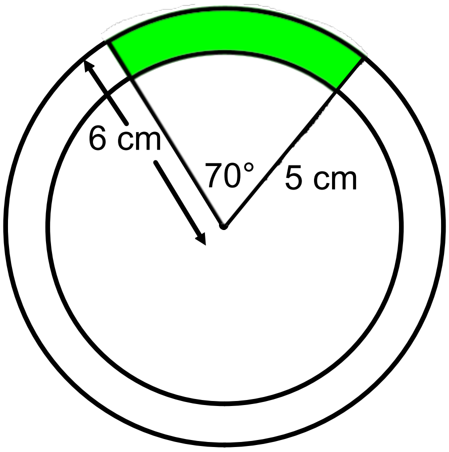
Warm-Up

Find the exact area of shaded section of the annulus.



Annulus

$\pi\cdot 6^2 - \pi\cdot 5^2$

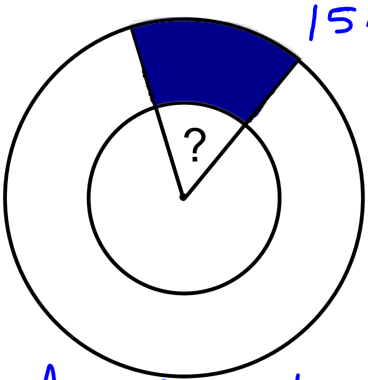
$36\pi - 25\pi = 11\pi$ cm²

$\frac{70}{360}\cdot 11\pi = 2\frac{5}{36}\pi$ cm²

or $2.13\overline{8}\,\pi$ cm²

$\frac{77}{36}\pi$ cm²

The area of the shaded region in the annulus is 15π in^2. If the radius of the larger circle is 12 in and the radius of the smaller circle is 6 in, what is the measure of the central angle?



$15\cancel{\pi} = \frac{c}{360}\cdot\cancel{\pi}\cdot 144 - \frac{c}{360}\,\cancel{\pi}\cdot 36$

$15 = \frac{144c}{360} - \frac{36c}{360}$

$15 = \frac{108c}{360}$

$5400 = 108c$

$50° = c$

Area of annulus

$\pi\cdot 12^2 - \pi\cdot 6^2 = 108\pi$

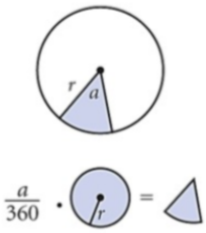
$\boxed{\frac{15\pi}{108\pi} = \frac{x}{360}}$

$\frac{x}{360}\cdot 108\cancel{\pi} = 15\cancel{\pi}$

$\frac{108x}{360} = 15$

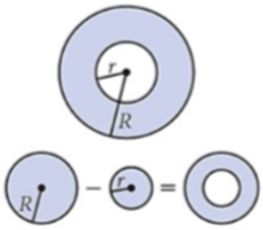
Recall:

A **sector of a circle** is the region between two radii and an arc of the circle.



$\frac{a}{360} \cdot \pi r^2 = A_{\text{sector}}$

An **annulus** is the region between two concentric circles.

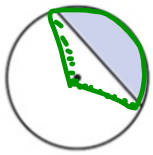


$\pi R^2 - \pi r^2 = A_{\text{annulus}}$

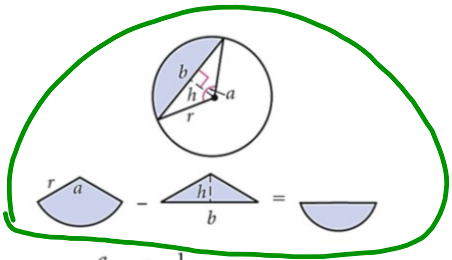
A **segment of a circle** is the region between a chord and an arc of the circle.

sector − triangle

$$A_{\text{segment}} = \frac{N}{360°} \cdot \pi r^2 - \frac{1}{2} bh$$



Segment of a circle

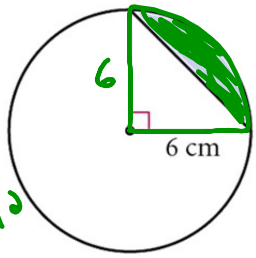


$\frac{a}{360} \pi r^2 - \frac{1}{2} bh = A_{\text{segment}}$

"Picture equations" are helpful when you try to visualize the areas of these regions. The picture equations below show you how to find the area of a sector of a circle, the area of a segment of a circle, and the area of an annulus.

Example 1

Find the area of the shaded segment.



$\frac{1}{4}$

$$A_{sector}: \frac{90}{360}\cdot \pi \cdot 6^2$$
$$= 9\pi \text{ cm}^2$$

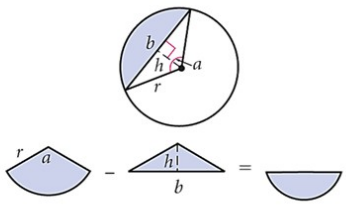
$$A_{triangle} = \frac{1}{2}\cdot 6\cdot 6 = 18 \text{ cm}^2$$

$$A_{segment} = \boxed{9\pi - 18 \text{ cm}^2}$$

or

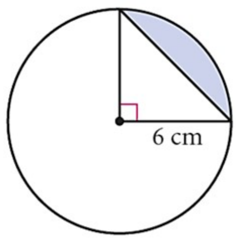
$$\boxed{10.3 \text{ cm}^2}$$

SOLUTION



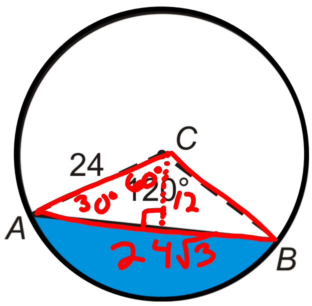
$$\frac{a}{360}\pi r^2 - \frac{1}{2}bh = A_{\text{segment}}$$

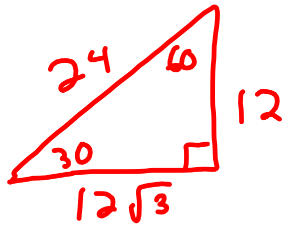
According to the picture equation on page 425, the area of a segment is equivalent to the area of the sector minus the area of the triangle. You can use the method in Example A to find that the area of the sector is $\frac{1}{4}\cdot 36\pi$ cm², or 9π cm². The area of the triangle is $\frac{1}{2}\cdot 6\cdot 6$, or 18 cm². So the area of the segment is $9\pi - 18$ cm².



Example 2

Find the area of shaded segment.





$A_{sector}: \frac{120}{360}\cdot\pi\cdot 24^2 = 192\pi$

$A_{triangle}: \frac{1}{2}\cdot 24\sqrt{3}\cdot 12 = 144\sqrt{3}$

$A_{segment}: \boxed{192\pi - 144\sqrt{3}\ u^2}$

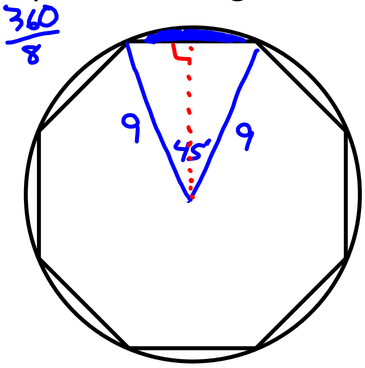
or

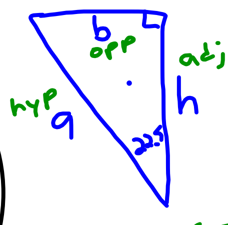
$\boxed{353.4\ u^2}$

Example 3

What is the area of <u>one</u> of the segments in the figure if the radius of the circle is 9 inches? (The octagon is regular.) Trigonometry

$\frac{360}{8}$





$A_{sector}: \frac{45}{360}\cdot\pi\cdot 9^2$

$= \frac{1}{8}\cdot 81\pi = \frac{81}{8}\pi$

or 10.125π

$\cos 22.5 = \frac{h}{9}$

$9\cos 22.5 = h$

$8.3149 \approx h$

$\sin 22.5 = \frac{b}{9}$

$9\sin 22.5 = b$

$3.4442 \approx b$

double this

$A_{triangle}$

$\frac{1}{2}\cdot 6.8884\cdot 8.3149$

$= 28.6382$

$A_{segment}$

$10.125\pi - 28.6382$

$3.17\ in^2$

Assignment: workbook p. 62 1-9