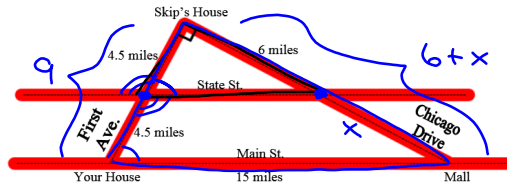


Warm Up

Chapter 7 – Similarity Transformations
Street Task

Name: _____
 Date: _____ Class: _____

Your friend Skip lives at the right angle of the right triangle formed by your house, his house and the mall. If State St. and Main St. run parallel to each other, answer the questions below.



Using your protractor, find the measure of the acute angle formed at Main and First.

62°

Using your protractor, find the measure of an obtuse angle formed at State and First.

118°

What should be true about the two angles you found above? Why is this the case?

How far is it from State St. to Main St. if you are traveling on Chicago Drive?

6 miles

What is the geometric term for State St. between First Ave. and Chicago Drive?

midsegment

If you have to pick Skip up before you go to the mall, how much longer will your drive be?

6 miles

Homework:

What questions do you have?



Proportional Segments Between Parallel Lines

Objectives

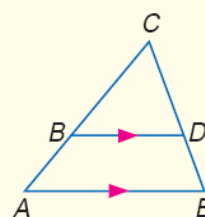
- Discover the relationship between the ratios of the parts into which parallel lines cut the sides of a triangle
- Extend the Parallel/Proportionality Conjecture to include multiple parallel lines

Parallel Lines and Proportional Parts

~~Theorem~~

Triangle Proportionality Theorem If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths.

Example: If $\overline{BD} \parallel \overline{AE}$, $\frac{BA}{CB} = \frac{DE}{CD}$.



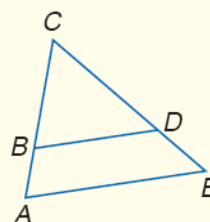
The converse of this theorem is also true:

~~Theorem~~

Converse of the Triangle Proportionality Theorem

If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

Example: If $\frac{BA}{CB} = \frac{DE}{CD}$, then $\overline{BD} \parallel \overline{AE}$.

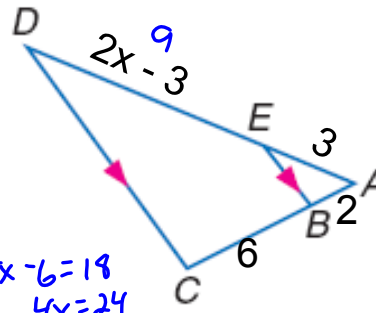


Example 1

Find x and ED if $AE = 3$, $AB = 2$, $BC = 6$, and $ED = 2x - 3$.

Solution: $\frac{2x-3}{6} = \frac{3}{2}$
 $\frac{2x-3}{3} = \frac{6}{2}$

$4x - 6 = 18$
 $4x = 24$
 $x = 6$



Example 2

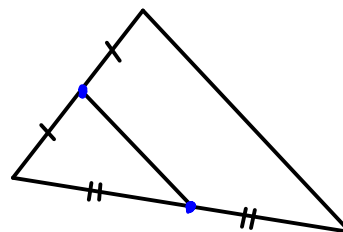
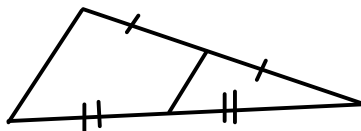
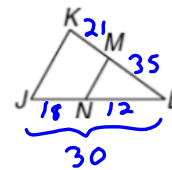
Determine whether $\overline{JK} \parallel \overline{NM}$.

$JN = 18$, $JL = 30$, $KM = 21$, and $ML = 35$

Solution:

~~$\frac{18}{30} = \frac{21}{35}$
 $630 = 630 \checkmark$~~

$\frac{18}{12} \stackrel{?}{=} \frac{21}{35}$
 $630 \neq 252$
 Not Parallel

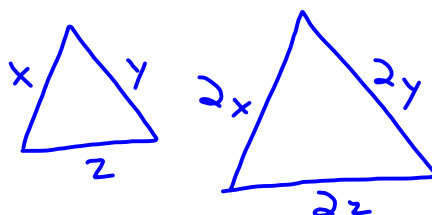
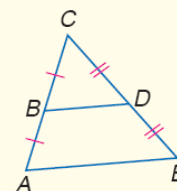


A **midsegment** of a triangle is a segment whose endpoints are the midpoints of two sides of the triangle.

Theorems

Triangle Midsegment Theorem A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.

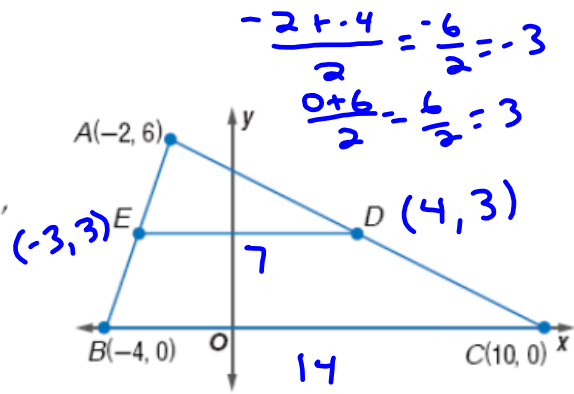
Example: If B and D are midpoints of \overline{AC} and \overline{EC} respectively, $\overline{BD} \parallel \overline{AE}$ and $BD = \frac{1}{2}AE$.



COORDINATE GEOMETRY For Exercises 6–8, use the following information.

Triangle ABC has vertices $A(-2, 6)$, $B(-4, 0)$, and $C(10, 0)$. \overline{DE} is a midsegment.

6. Find the coordinates of D and E .
7. Verify that \overline{DE} is parallel to \overline{BC} .
8. Verify that $DE = \frac{1}{2}BC$.



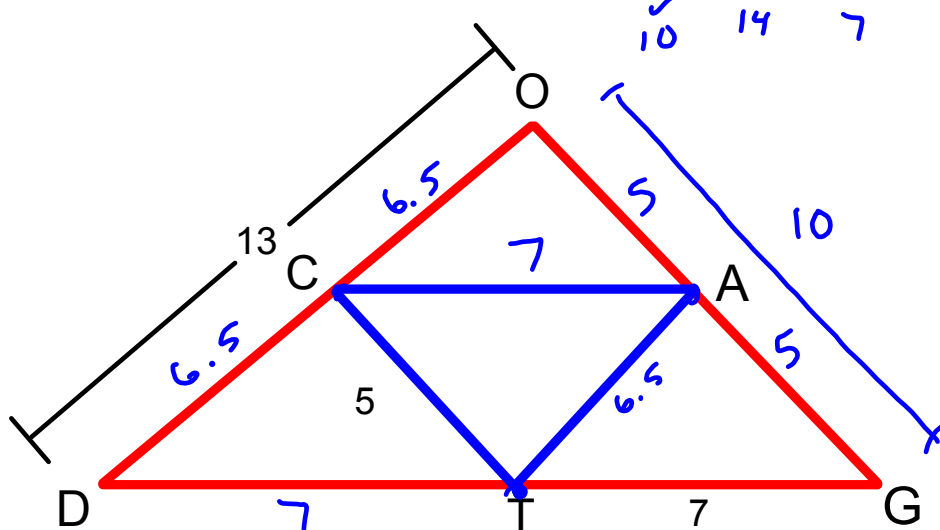
Solutions:

6.
7.
8.

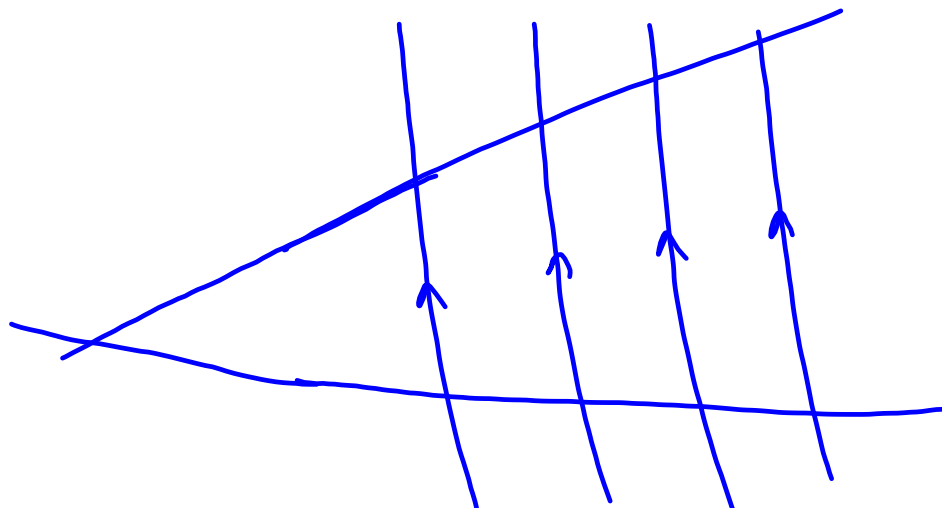
Example 3

\overline{CA} , \overline{AT} , and \overline{CT} are midsegments of $\triangle DOG$.

$CT = 5$, $TG = 7$, and $DO = 13$. Find GO , DG , CA , and AT .



Solution:

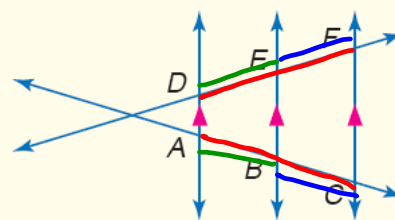


Corollaries

- 1** If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

Example: If $\overline{DA} \parallel \overline{EB} \parallel \overline{FC}$, then $\frac{AB}{BC} = \frac{DE}{EF}$

$$\frac{AC}{DF} = \frac{BC}{EF} \text{ and } \frac{AC}{BC} = \frac{DF}{EF}$$

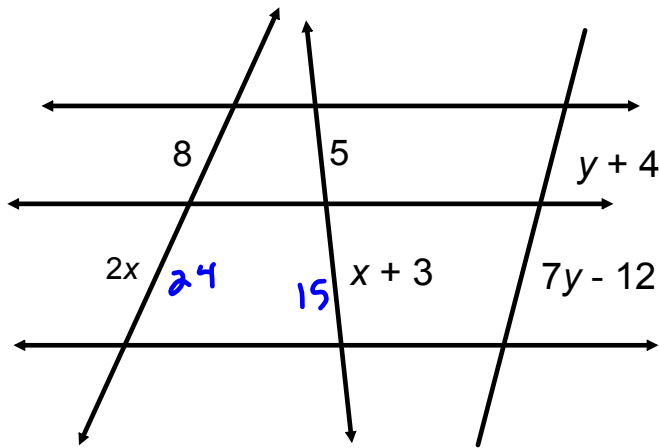


- 2** If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

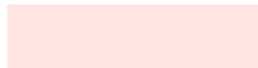
Example: If $\overline{AB} \cong \overline{BC}$, then $\overline{DE} \cong \overline{EF}$.

Example 4

Find x and y if all the horizontal lines are parallel



Solutions:



$$\frac{8}{5} = \frac{2x}{x+3}$$

$$8x + 24 = 10x$$

$$24 = 2x$$

$$12 = x$$

$$\frac{8}{24} = \frac{y+4}{7y-12}$$

$$56y - 96 = 24y + 96$$

$$32y = 192$$

$$x = 6$$

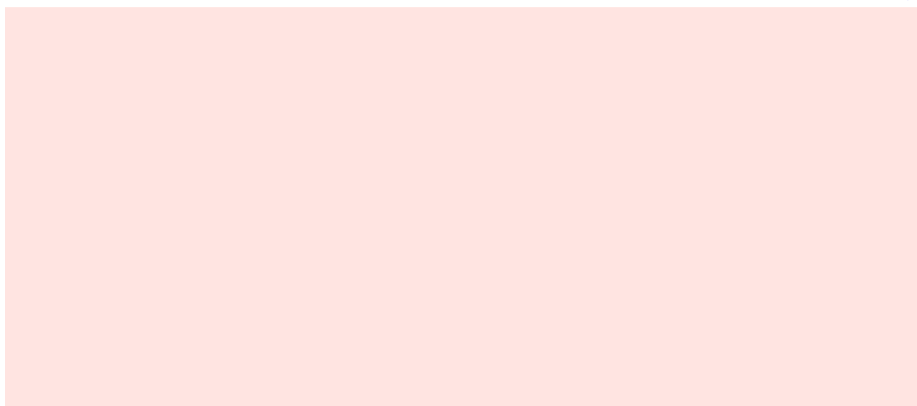
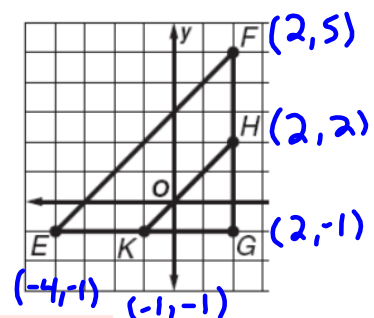
Example 5

Triangle EFG has vertices $E(-4, -1)$, $F(2, 5)$, and $G(2, -1)$. Point K is the midpoint of \overline{EG} and H is the midpoint of \overline{FG} .

Show that \overline{EF} is parallel to \overline{KH} .

$$\text{Slope of } \overline{FE} = \frac{5+1}{2+4} = \frac{6}{6} = 1$$

$$\text{Slope of } \overline{HK} = \frac{2+1}{2+1} = \frac{3}{3} = 1$$



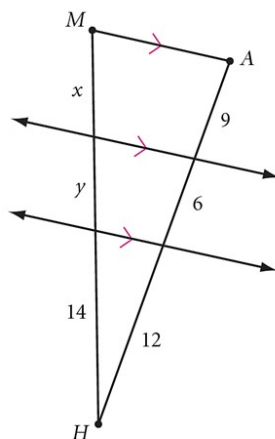


Proportional Segments Between Parallel Lines

Extra Example

$x = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}}$



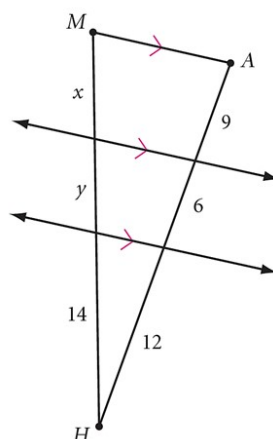
Proportional Segments Between Parallel Lines

Extra Example

ANSWER

$x = 10.5$

$y = 7$



Homework:

Textbook p. 400-403: 1-12; 19-20

Learning Targets

7.5 Proportional Segments

A. I can use the relationship between the ratios of the parts into which parallel lines cut the sides of a triangle

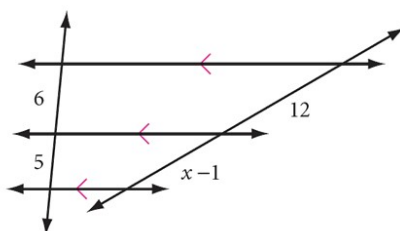
B. I can extend the Parallel/Perpendicular Conjecture to include multiple parallel lines



Proportional Segments Between Parallel Lines

Closing Question

Solve for x .



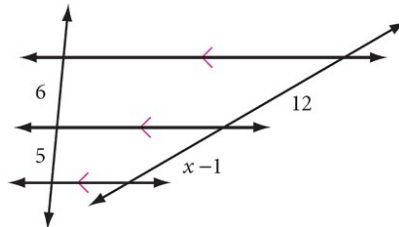


Proportional Segments Between Parallel Lines

Closing Question

ANSWER

$x = 11$



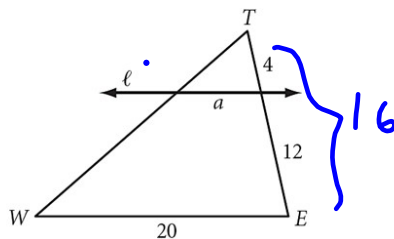
7.5 Exercises

pages 400 – 403

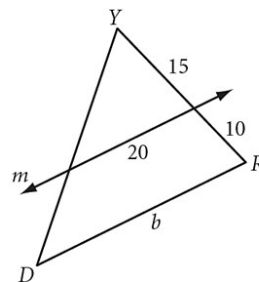
For Exercises 1–12, all measurements are in centimeters.

1. $\ell \parallel \overline{WE}$
 $a = ?$ (h)

$\frac{4}{16} = \frac{a}{20}$

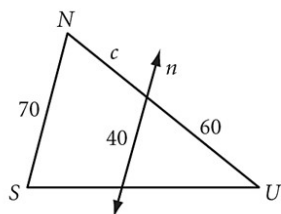


2. $m \parallel \overline{DR}$
 $b = ?$

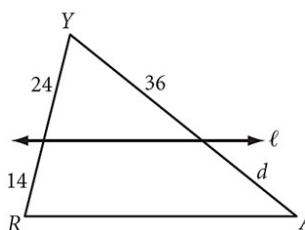


7.5 Exercises
pages 400 – 403

3. $n \parallel \overline{SN}$
 $c = ?$ (h)

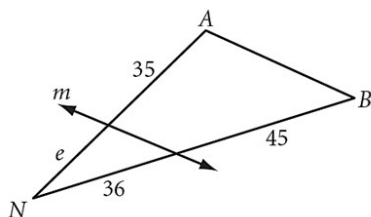


4. $\ell \parallel \overline{RA}$
 $d = ?$

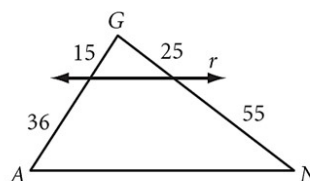


7.5 Exercises
pages 400 – 403

5. $m \parallel \overline{BA}$
 $e = ?$

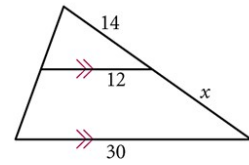


6. Is $r \parallel \overline{AN}$? (h)



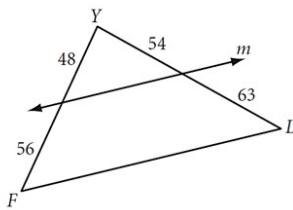
7.5 Exercises
pages 400 – 403

7. Alex and José have each found the value of x in the diagram at right in different ways and are explaining their methods. Alex says, “The proportion I used to solve this problem is $\frac{x}{14} = \frac{30}{12}$, so x is 35.” José looks puzzled and responds, “I got x is 21 by using the proportion $\frac{x+14}{30} = \frac{14}{12}$.” Explain which method is correct and what is wrong with the incorrect method.

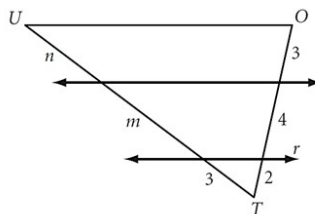


7.5 Exercises
pages 400 – 403

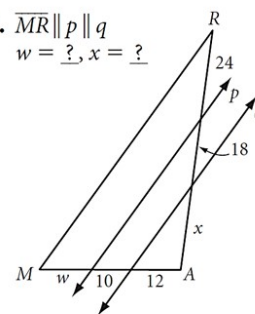
8. Is $m \parallel \overline{FL}$?



9. $r \parallel s \parallel \overline{OU}$
 $m = ?$, $n = ?$

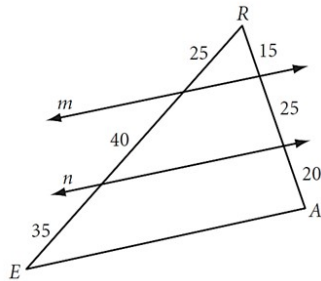


10. $\overline{MR} \parallel p \parallel q$
 $w = ?$, $x = ?$

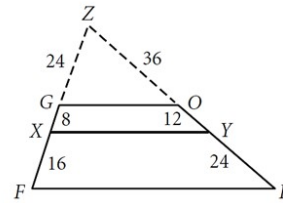


7.5 Exercises
pages 400 – 403

11. Is $m \parallel \overline{EA}$?
Is $n \parallel \overline{EA}$?
Is $m \parallel n$?



12. Is $\overline{XY} \parallel \overline{GO}$?
Is $\overline{XY} \parallel \overline{FR}$?
Is $FROG$ a trapezoid?

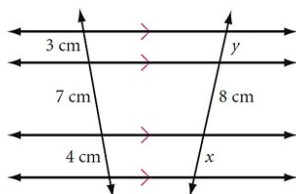


13. Draw segment EF . Use compass and straightedge to divide it into five equal parts.

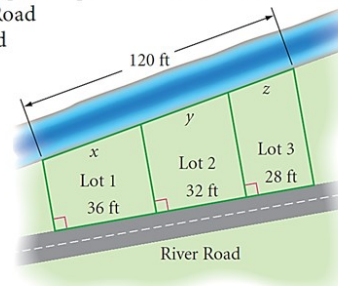
7.5 Exercises
pages 400 – 403

The Extended Parallel/Proportionality Conjecture can be extended even further. That is, you don't necessarily need a triangle. If three or more parallel lines intercept two other lines (transversals) in the same plane, they do so proportionally. For Exercises 19 and 20 use this extension.

19. Find x and y .



20. A real estate developer has parceled land between a river and River Road as shown. The land has been divided by segments perpendicular to the road. What is the "river frontage" (lengths x , y , and z) for each of the three lots?

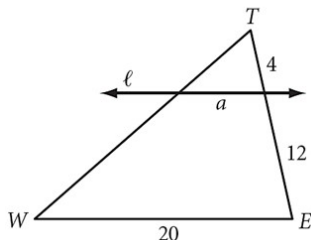


ANSWERS

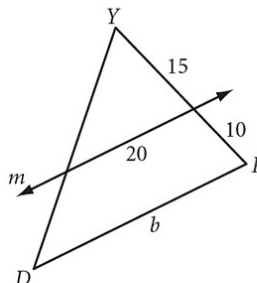
7.5 Exercises

For Exercises 1–12, all measurements are in centimeters.

1. $\ell \parallel \overline{WE}$
 $a = ?$ **5 cm**



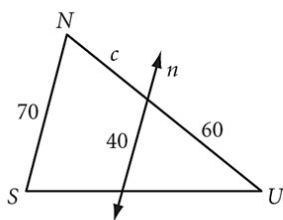
2. $m \parallel \overline{DR}$
 $b = ?$ **$33\frac{1}{3}$ cm**



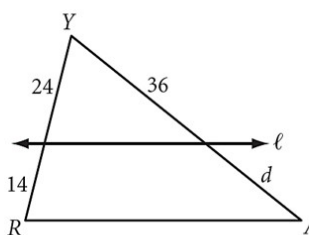
ANSWERS

7.5 Exercises

3. $n \parallel \overline{SN}$
 $c = ?$ **45 cm**



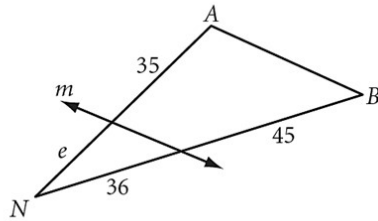
4. $\ell \parallel \overline{RA}$
 $d = ?$ **21 cm**



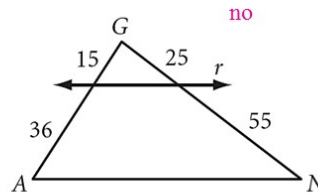
ANSWERS

7.5
Exercises

5. $m \parallel \overline{BA}$
 $e = ?$ 28 cm



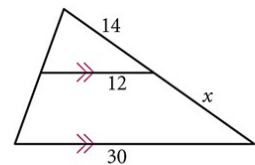
6. Is $r \parallel \overline{AN}$? h



ANSWERS

7.5
Exercises

7. Alex and José have each found the value of x in the diagram at right in different ways and are explaining their methods. Alex says, “The proportion I used to solve this problem is $\frac{x}{14} = \frac{30}{12}$, so x is 35.” José looks puzzled and responds, “I got x is 21 by using the proportion $\frac{x+14}{30} = \frac{14}{12}$.” Explain which method is correct and what is wrong with the incorrect method.

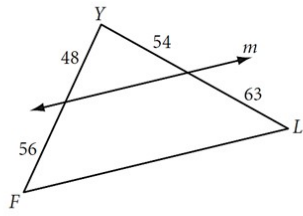


José’s method is correct. Possible explanation: Alex’s first ratio compares only part of a side of the larger triangle to the entire corresponding side of the smaller triangle, while the second ratio compares entire corresponding sides of the triangles.

ANSWERS

7.5 Exercises

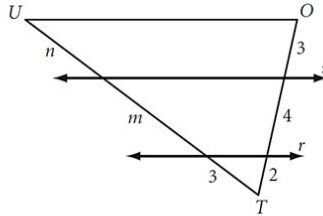
8. Is $m \parallel \overline{FL}$?



Yes

9. $r \parallel s \parallel \overline{OU}$

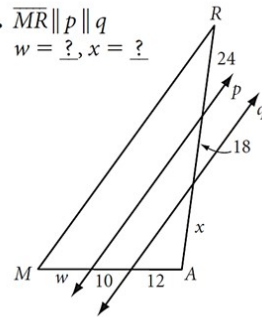
$m = ?$, $n = ?$



$m = 6 \text{ cm}$, $n = 4.5 \text{ cm}$

10. $\overline{MR} \parallel p \parallel q$

$w = ?$, $x = ?$

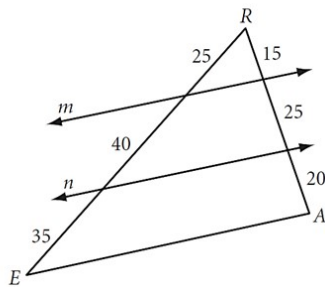


$w = 13.\overline{3} \text{ cm}$, $x = 21.6 \text{ cm}$

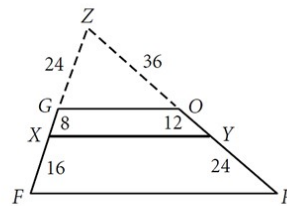
ANSWERS

7.5 Exercises

11. Is $m \parallel \overline{EA}$? **yes**
 Is $n \parallel \overline{EA}$? **no**
 Is $m \parallel n$? **no**



12. Is $\overline{XY} \parallel \overline{GO}$? **yes**
 Is $\overline{XY} \parallel \overline{FR}$? **yes**
 Is $FROG$ a trapezoid? **yes**



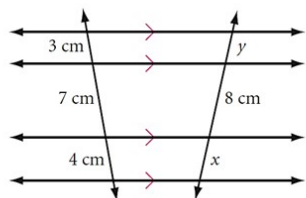
ANSWERS

7.5

Exercises

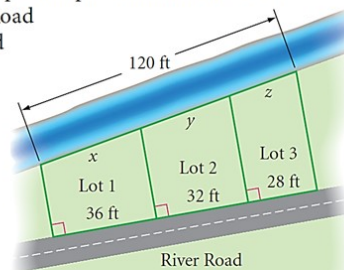
The Extended Parallel/Proportionality Conjecture can be extended even further. That is, you don't necessarily need a triangle. If three or more parallel lines intercept two other lines (transversals) in the same plane, they do so proportionally. For Exercises 19 and 20 use this extension.

19. Find x and y .



$$x \approx 4.6 \text{ cm}, y \approx 3.4 \text{ cm}$$

20. A real estate developer has parceled land between a river and River Road as shown. The land has been divided by segments perpendicular to the road. What is the "river frontage" (lengths x , y , and z) for each of the three lots?



$$x = 45 \text{ ft}, y = 40 \text{ ft}, z = 35 \text{ ft}$$