Discuss with your partner -- use the Thursday section of your warm-up sheet!

How are similar figures different from congruent figures?

Use examples to explain your answer. (use pictures if desired)

ng

How are similar figures different from congruent figures?

The lengths of the sides of congruent figures are equal. The lengths of the sides of similar figures are proportional.

Use examples to explain your answer.

Examples will vary.

Lesson 7.2: Similar Triangles

LESSON 7.1 · Similar Polygons

1. $AP = 8$ cm; $EI = 7$ cm; $SN = 15$ cm; $YR = 12$ cm

2. $SL = 5.2$ cm; $MI = 10$ cm; $m\angle D = 120°$;
   $m\angle U = 85°$; $m\angle A = 80°$

3. Yes. All corresponding angles are congruent. Both figures are parallelograms, so opposite sides within each parallelogram are equal. The corresponding sides are proportional $\left(\frac{15}{5} = \frac{9}{3}\right)$.

4. Yes. Corresponding angles are congruent by the CA Conjecture. Corresponding sides are proportional $\left(\frac{2}{4} = \frac{3}{6} = \frac{4}{8}\right)$.

5. No. $\frac{6}{18} \neq \frac{8}{22}$.

6. Yes. All angles are right angles, so corresponding angles are congruent. The corresponding side lengths have the ratio $\frac{4}{7}$, so corresponding side lengths are proportional.

LESSON

7.2

Similar Triangles

Objectives

- Discover shortcut methods for determining similar triangles

7.2 Similar Triangles: Learning Target
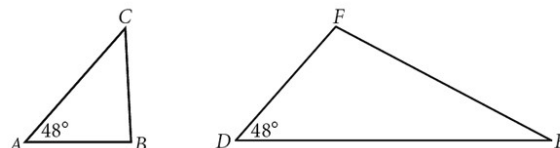A. I can use shortcut methods for determine triangles are similar (AA, SSS, SAS)

LESSON 7.2

Similar Triangles

In Lesson 7.1, you concluded that you must know about both the angles and the sides of two quadrilaterals in order to make a valid conclusion about their similarity.

However, triangles are unique. Recall from Chapter 4 that you found four shortcuts for triangle congruence: SSS, SAS, ASA, and SAA. Are there shortcuts for triangle similarity as well? Let's first look for shortcuts using only angles.

The figures below illustrate that you cannot conclude that two triangles are similar given that only one set of corresponding angles is congruent. How about two sets of congruent angles?
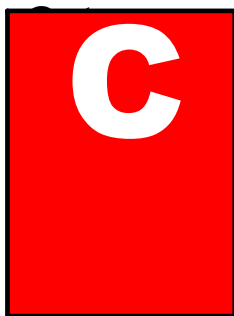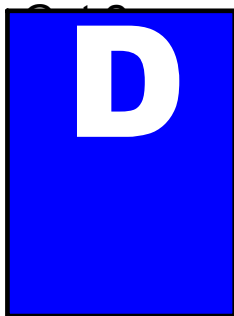

$\angle A \cong \angle D$, but $\triangle ABC$ is not similar to $\triangle DEF$ or to $\triangle DFE$.

Lesson 7.2: Similar Triangles

What are the 5 ways to show that triangles are congruent?

Turn into groups and decide who will be the following:
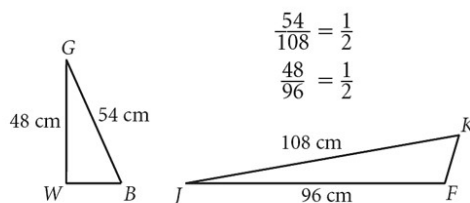
D C H S

LESSON
7.2

Similar Triangles

Now let's look for shortcuts for similarity that use only sides. The figures at right illustrate that you cannot conclude that two triangles are similar given that two sets of corresponding sides are proportional.

$\frac{GB}{JK} = \frac{GW}{JF}$, but $\triangle GWB$ is not similar to $\triangle JFK$.

How about all three sets of corresponding sides?

$\frac{54}{108} = \frac{1}{2}$

$\frac{48}{96} = \frac{1}{2}$

G

48 cm   54 cm

W   B   J   108 cm   K   96 cm   F

INVESTIGATION 2

Is SSS a Similarity Shortcut?

YOU WILL NEED:
compass,
straightedge,
protractor,
patty paper

If three sides of one triangle are proportional to the three sides of another triangle, must the two triangles be similar?

Draw any triangle *ABC*. Then construct a second triangle, *DEF*, whose side lengths are a multiple of the original triangle. (Your second triangle can be larger or smaller.) Compare the corresponding angles of the two triangles.

Compare your results with the results of others near you and state a conjecture.

SSS Similarity Conjecture	**C-60**
If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are _____ .	

INVESTIGATION 2
SOLUTION

If three sides of one triangle are proportional to the three sides of another triangle, must the two triangles be similar?

Draw any triangle *ABC*. Then construct a second triangle, *DEF*, whose side lengths are a multiple of the original triangle. (Your second triangle can be larger or smaller.) Compare the corresponding angles of the two triangles.

Compare your results with the results of others near you and state a conjecture.

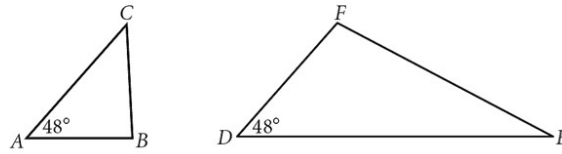Corresponding angles should be congruent.

SSS Similarity Conjecture	**C-60**
If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar.	

However, triangles are unique. Recall from Chapter 4 that you found four shortcuts for triangle congruence: SSS, SAS, ASA, and SAA. Are there shortcuts for triangle similarity as well? Let's first look for shortcuts using only angles.

The figures below illustrate that you cannot conclude that two triangles are similar given that only one set of corresponding angles is congruent. How about two sets of congruent angles?

$\angle A \cong \angle D$, but $\triangle ABC$ is not similar to $\triangle DEF$ or to $\triangle DFE$.

INVESTIGATION 1

Is AA a Similarity Shortcut?

If two angles of one triangle are congruent to two angles of another triangle, must the two triangles be similar?

Step 1 Draw any triangle ABC.

Step 2 Construct a second triangle, DEF, with $\angle D \cong \angle A$ and $\angle E \cong \angle B$. What will be true about $\angle C$ and $\angle F$? Why?

Step 3 Carefully measure the lengths of the sides of both triangles. Compare the ratios of the corresponding sides. Is $\frac{AB}{DE} \approx \frac{AC}{DF} \approx \frac{BC}{EF}$?

Step 4 Compare your results with the results of others near you. You should be ready to state a conjecture.

AA Similarity Conjecture	**C-59**
If _____ angles of one triangle are congruent to _____ angles of another triangle, then _____ .	

INVESTIGATION 1
SOLUTION

Step 2 Construct a second triangle, *DEF*, with $\angle D \cong \angle A$ and $\angle E \cong \angle B$. What will be true about $\angle C$ and $\angle F$? Why?

∠F must be congruent to ∠C because the three angles of each triangle must add up to 180°.

Step 3 Carefully measure the lengths of the sides of both triangles. Compare the ratios of the corresponding sides. Is $\frac{AB}{DE} \approx \frac{AC}{DF} \approx \frac{BC}{EF}$?

The ratios should be equal.

Step 4 Compare your results with the results of others near you. You should be ready to state a conjecture.

AA Similarity Conjecture	**C-59**
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.	

So SSS and AA are shortcuts for triangle similarity. That leaves SAS and SSA as possible shortcuts to consider.

INVESTIGATION 3

YOU WILL NEED:
compass,
straightedge,
patty paper

Is SAS a Similarity Shortcut?

Is SAS a shortcut for similarity? Try to construct two different triangles that are not similar but have two pairs of sides proportional and the pair of included angles equal in measure.

Compare the measures of corresponding sides and corresponding angles. Share your results with others near you and state a conjecture.

SAS Similarity Conjecture	**C-61**
If two sides of one triangle are proportional to two sides of another triangle and _____, then the _____.	

INVESTIGATION 3
SOLUTION

Is SAS a shortcut for similarity? Try to construct two different triangles that are not similar but have two pairs of sides proportional and the pair of included angles equal in measure.

Corresponding sides should be proportional, and corresponding angles should be congruent.

Compare the measures of corresponding sides and corresponding angles. Share your results with others near you and state a conjecture.
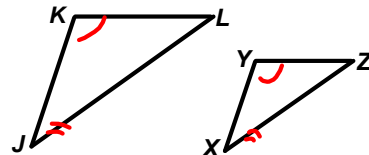
SAS Similarity Conjecture   **C-61**

If two sides of one triangle are proportional to two sides of another triangle and the included angles are congruent, then the triangles are similar.

Postulate : Angle-Angle Similarity (AA~)

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

If $\angle K \cong \angle Y$ and $\angle J \cong \angle X$, then $\triangle JKL \sim \triangle XYZ$

Theorem : Side - Side - Side Similarity (SSS~)

If the corresponding sides of two triangles are proportional, then the triangles are similar.
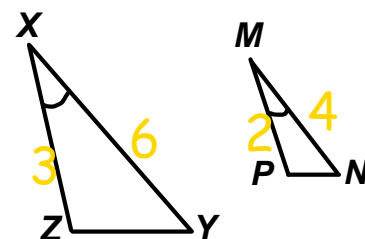
If $\dfrac{AB}{FG} = \dfrac{BC}{GH} = \dfrac{CA}{HF}$, then $\triangle ABC \sim \triangle FGH$.
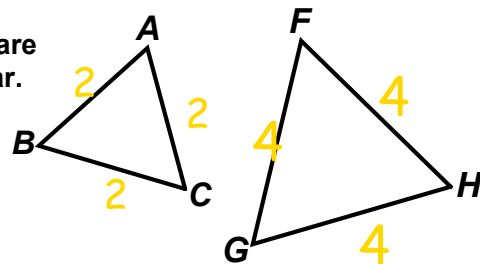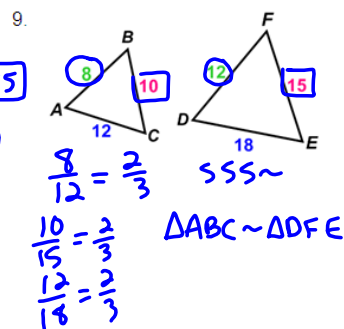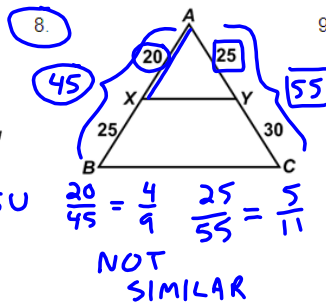
Theorem : Side - Angle - Side Similarity (SAS~)
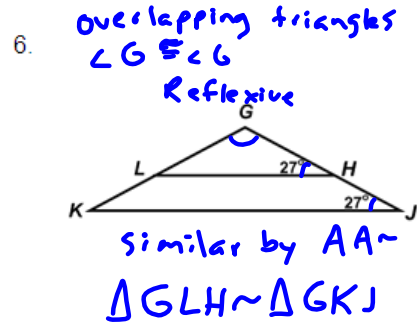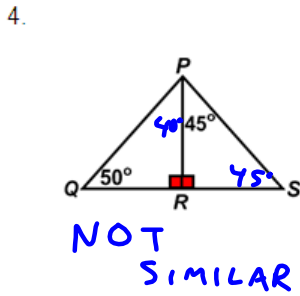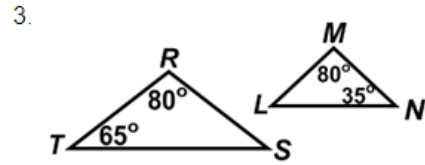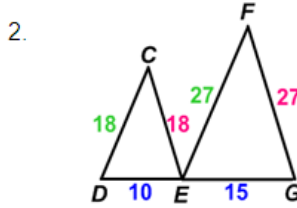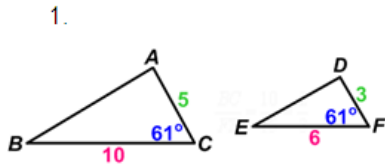
If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides that include these angles are proportional, then the triangles are similar.
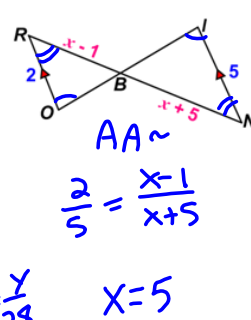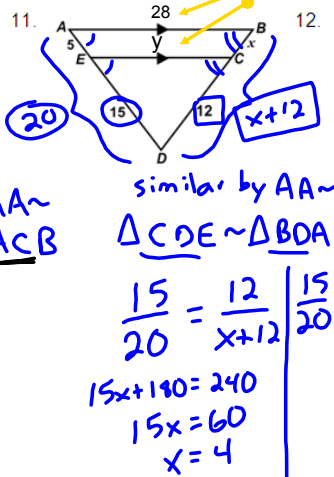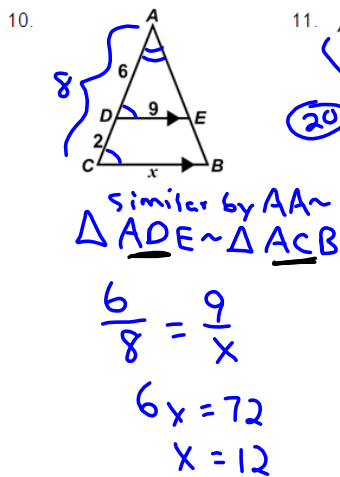
If $\angle X \cong \angle M$ and $\dfrac{XZ}{MP} = \dfrac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Determine whether the triangles are similar. If they are similar, write a similarity statement.

1.

A
5
B   61° C
10

D
3
E   61° F
6

2.

F
C   27   27
18   18
D 10 E 15 G

3.

R
80°
T 65° S

M
80°
L   35° N

4.

P
60° 45°
Q 50°   75° S
R

NOT
SIMILAR

5.

J
10
K   6   L

R
4
S   3   T

$\frac{10}{4} = \frac{5}{2}$ **NOT**
SIMILAR

$\frac{6}{3} = 2$

6.

overlapping triangles
∠G ≅ ∠G
Reflexive

G
L   27°   H
K   27°   J

similar by AA~
ΔGLH~ΔGKJ

7.

Q
R   T
S   U

AA~
ΔQRT~ΔQSU

8.

A
20   25
45   X   Y   55
25   30
B   C

$\frac{20}{45} = \frac{4}{9}$ $\frac{25}{55} = \frac{5}{11}$

NOT
SIMILAR

9.

B
8   10
A 12 C

F
12   15
D 18 E

$\frac{8}{12} = \frac{2}{3}$ SSS~

$\frac{10}{15} = \frac{2}{3}$ ΔABC~ΔDFE

$\frac{12}{18} = \frac{2}{3}$
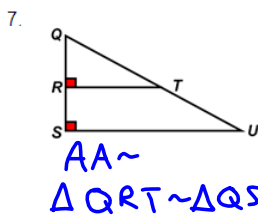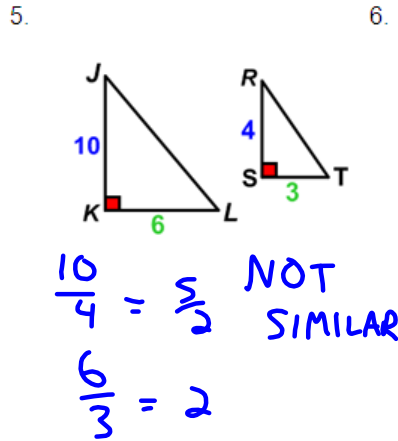
Explain how the triangles are similar. Then find the value of x.   **ADD**

10.

A
6
D   9   E
2
C   x   B

Similar by AA~
ΔADE~ΔACB

$\frac{6}{8} = \frac{9}{x}$

$6x = 72$

$X = 12$

11.

28
A   B
5   E   Y   x
C
20   15   12   x+12
D

similar by AA~
ΔCDE~ΔBDA

$\frac{15}{20} = \frac{12}{x+12}$ $\frac{15}{20} = \frac{y}{28}$

$15x+180=240$

$15x=60$

$x=4$

12.

R   x-1   I
2   5
O   B   x+5   N

AA~

$\frac{2}{5} = \frac{x-1}{x+5}$

$X=5$

Similar Triangles

Extra Example

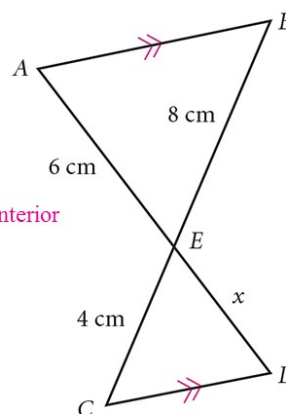Why is $\triangle ABE \sim \triangle DCE$?

Find x.
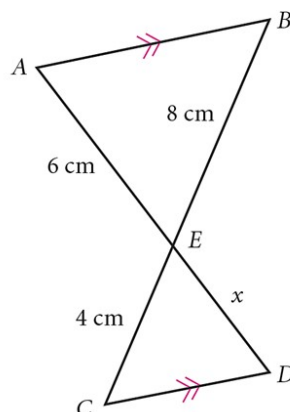
Similar Triangles

Extra Example
ANSWER

Why is $\triangle ABE \sim \triangle DCE$?

Since $\overline{AB} \parallel \overline{CD}$, $\angle A \cong \angle D$ and $\angle B \cong \angle C$ by the Alternate Interior Angles Conjecture. They are similar by the AA Similarity Conjecture.

Find x. $x = 2$ cm.

Homework:

Similar Triangles Packet

Finish examples on notes

Quiz Tomorrow
(Friday) on 7.1-7.2