

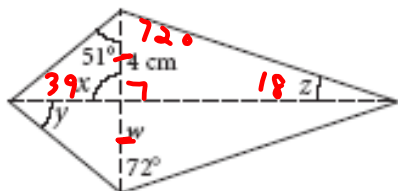
Warm-Up: Find the missing variable values.

$$w = \frac{4}{7} \quad x = \frac{90^\circ}{18}$$

$$y = \frac{39^\circ}{7} \quad z = \frac{18}{7}$$

$$x = \frac{61^\circ}{7}$$

$$y = \frac{119^\circ}{7}$$



Learning Targets:

5.7 Properties of Midsegments

- I can define a midsegment.
- Given a midsegment, I can write and solve equations.



Properties of Midsegments

Objectives

- Define and discover properties of midsegments in triangles and trapezoids



Properties of Midsegments

Launch

Draw a triangle on patty paper. Construct the midpoint of two sides of the triangle. Connect these two points, creating the midsegment. How many midsegments does a triangle have in total?



Properties of Midsegments

Launch

3 midsegments

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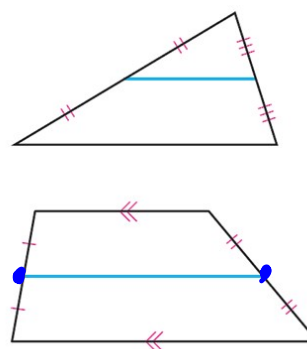
Lesson 5.7 Properties of Midsegments



Properties of Midsegments

As you learned in Chapter 3, the segment connecting the midpoints of two sides of a triangle is a midsegment of the triangle. The segment connecting the midpoints of the two nonparallel sides of a trapezoid is also called the midsegment of the trapezoid.

In this lesson you will discover special properties of midsegments.



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Lesson 5.7 Properties of Midsegments

INVESTIGATION 1

YOU WILL NEED:
patty paper,
straightedge

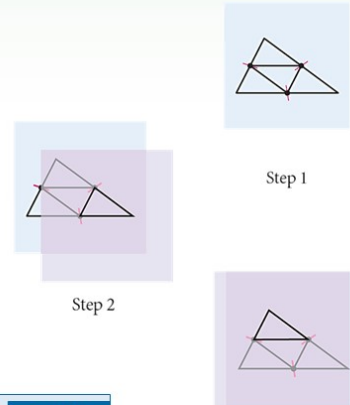
Triangle Midsegment Properties

In this investigation you will discover two properties of triangle midsegments. Each person in your group can investigate a different triangle.

Step 1 Draw a triangle on a piece of patty paper. Pinch the patty paper to locate midpoints of the sides. Draw the midsegments. You should now have four small triangles.

Step 2 Place a second piece of patty paper over the first and copy one of the four triangles.

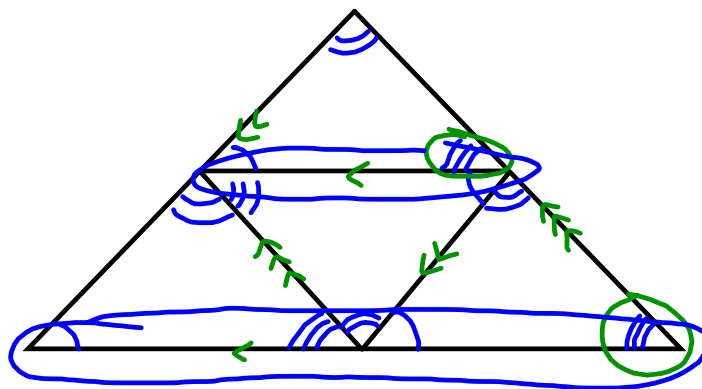
Step 3 Compare all four triangles by sliding the copy of one small triangle over the other three triangles. Compare your results with the results of your group. Copy and complete the conjecture.



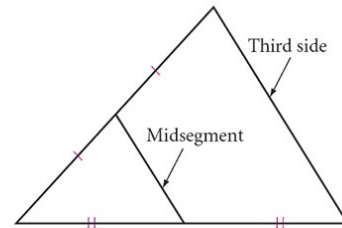
Three Midsegments Conjecture

C-50

The three midsegments of a triangle divide it into _____.



Step 4 Mark all the congruent angles on the original patty paper. If you find it too cluttered, redraw the original triangle on regular paper with just one midsegment, as in the diagram at right, and then mark all the congruent angles. Using the Corresponding Angles Conjecture or its converse, what conclusions can you make about a midsegment and the large triangle's third side?



Step 5 Compare the length of the midsegment to the large triangle's third side. How do they relate? Copy and complete the conjecture

Triangle Midsegments Conjecture

C-51

A midsegment of a triangle is _____ to the third side and _____ the length of _____.

In the next investigation you will discover two properties of the midsegment of a trapezoid.



INVESTIGATION 1 SOLUTION

Three Midsegments Conjecture

C-50

The three midsegments of a triangle divide it into **four congruent triangles**.

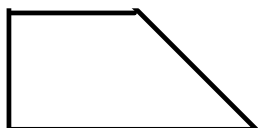
Step 4 Mark all the congruent angles on the original patty paper. If you find it too cluttered, redraw the original triangle on regular paper with just one midsegment, as in the diagram at right, and then mark all the congruent angles. Using the Corresponding Angles Conjecture or its converse, what conclusions can you make about a midsegment and the large triangle's third side?

Each midsegment is parallel to the third side.

Triangle Midsegments Conjecture

C-51

A midsegment of a triangle is **parallel** to the third side and **half** the length of **the third side**.



INVESTIGATION 2

YOU WILL NEED:
patty paper,
straightedge

Trapezoid Midsegment Properties

Each person in your group can investigate a different trapezoid. Make sure you draw the two bases perfectly parallel.

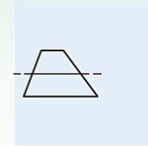
Step 1 Draw a small trapezoid on the left side of a piece of patty paper. Pinch the paper to locate the midpoints of the nonparallel sides. Draw the midsegment.

Step 2 Label the angles as shown. Place a second piece of patty paper over the first and copy the trapezoid and its midsegment.

Step 3 Compare the trapezoid's base angles with the corresponding angles at the midsegment by sliding the copy up over the original.

Step 4 Are the corresponding angles congruent? What can you conclude about the midsegment and the bases? Compare your results with the results of other students.

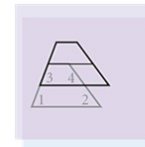
The midsegment of a triangle is half the length of the third side. How does the length of the midsegment of a trapezoid compare to the lengths of the two bases? Let's investigate.



Step 1



Step 2

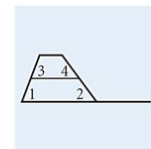


Step 3

Lesson 5.7 Properties of Midsegments

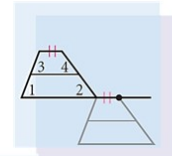
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Step 5 On the original trapezoid, extend the longer base to the right by at least the length of the shorter base.



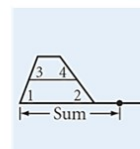
Step 5

Step 6 Slide the second patty paper under the first. Show the sum of the lengths of the two bases by marking a point on the extension of the longer base.



Step 6

Step 7 How many times does the midsegment fit onto the segment representing the sum of the lengths of the two bases? What do you notice about the length of the midsegment and the sum of the lengths of the two bases?



Step 8 Combine your conclusions from Steps 4 and 7 and complete this conjecture.

Trapezoid Midsegment Conjecture

C-52

A midsegment of a trapezoid is _____ to the bases and is equal in length to _____.

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Lesson 5.7 Properties of Midsegments



INVESTIGATION 2

SOLUTION

Step 4 Are the corresponding angles congruent? What can you conclude about the midsegment and the bases? Compare your results with the results of other students.

Yes; the midsegment is parallel to the base.

Step 7 How many times does the midsegment fit onto the segment representing the sum of the lengths of the two bases? What do you notice about the length of the midsegment and the sum of the lengths of the two bases?

twice; half the sum of the lengths of the bases

Trapezoid Midsegment Conjecture

C-52

A midsegment of a trapezoid is parallel to the bases and is equal in length to the average of the length of the bases.

(half the sum of the two bases)

LESSON

5.7

Properties of Midsegments

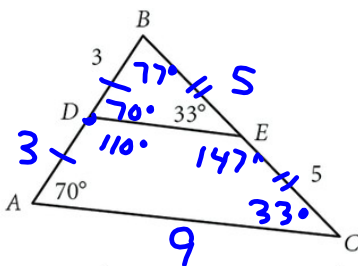
What happens if one base of the trapezoid shrinks to a point? Then the trapezoid collapses into a triangle, the midsegment of the trapezoid becomes a midsegment of the triangle, and the Trapezoid Midsegment Conjecture becomes the Triangle Midsegment Conjecture. Do both of your midsegment conjectures work for the last figure?



Properties of Midsegments

Extra Example

25-6-10



\overline{DE} is a midsegment. The measurements are in meters.

Find all the missing angles in the figure.

If the perimeter is 25 m, $AC = ?$ Explain your reasoning.

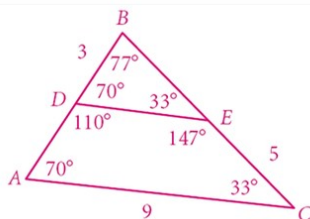


Properties of Midsegments

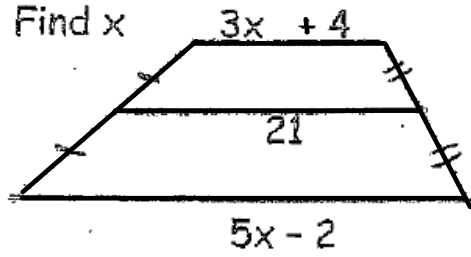
Extra Example

ANSWER

$AC = 9\text{m}$



3) Find x



Midsegment = $\frac{1}{2}$ of sum of bases

$$\frac{3x + 4 + 5x - 2}{2} = 21$$

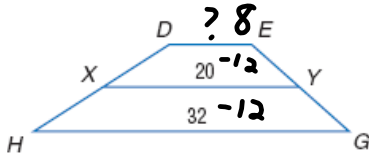
$$42 = 3x + 4 + 5x - 2$$

$$42 = 8x + 2$$

$$40 = 8x$$

$$5 = x$$

For trapezoid $DEGH$, X and Y are midpoints of the legs. Find DE . ★



$$20 \cdot 2 = 40$$

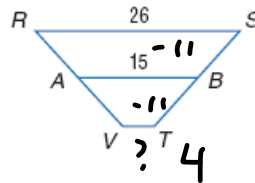
$$40 - 32 = 8$$

$$\frac{x + 32}{2} = 20$$

$$x + 32 = 40$$

$$x = 8$$

For trapezoid $RSTV$, A and B are midpoints of the legs. Find VT . ★



$$15 \cdot 2 = 30$$

$$\frac{30 - 26}{4}$$

Homework: Workbook 5.7