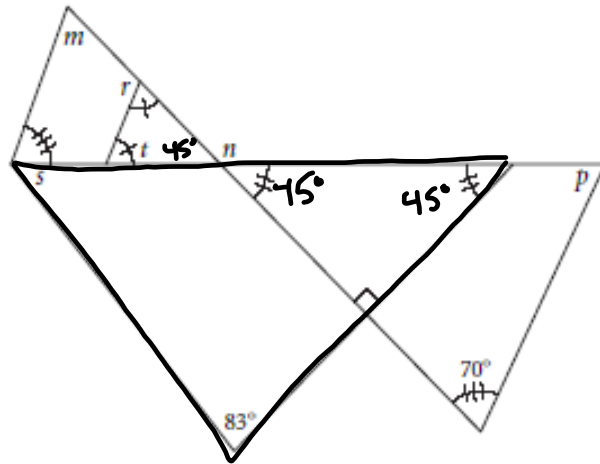


Warm-Up: Find the measure of the following angles.

$$\begin{aligned} m &= \underline{65} \\ n &= \underline{135} \\ p &= \underline{65} \\ r &= \underline{112.5} \\ s &= \underline{52} \\ t &= \underline{67.5} \end{aligned}$$



Learning Targets:

5.4 Properties of Parallelograms

- I can identify the properties of a parallelogram.
- Given a parallelogram, I can write and solve equation based on their properties.

5.5 Properties of Special Parallelograms

- I can identify the properties of a special parallelogram: rectangle, rhombus, and square.
- Given a rectangle, rhombus, or square, I can write and solve equations based on their properties.



Properties of Parallelograms

In this lesson you will discover some special properties of parallelograms. A parallelogram is a quadrilateral whose opposite sides are parallel.

Rhombuses, rectangles, and squares all fit this definition as well. Therefore, any properties you discover for parallelograms will also apply to these other shapes. However, to be sure that your conjectures will apply to *any* parallelogram, you should investigate parallelograms that don't have any other special properties such as right angles, all congruent angles, or all congruent sides.

Please turn into table groups and get the following supplies:

- ruler or protractor
- piece of graph paper
- worksheet - Investigation

Working with your table group, follow instructions and complete Investigation 1 and check answers with key. Next, complete Investigation 2 and check answers.



When both are finished, you will receive practice worksheet.

 **INVESTIGATION**

Four Parallelogram Properties

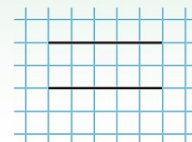
YOU WILL NEED:

graph paper, patty paper or a compass, a double-edged straightedge, a protractor

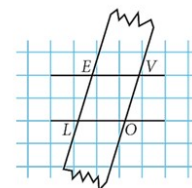
First you'll create a parallelogram.

Step 1 Using the lines on a piece of graph paper as a guide, draw a pair of parallel lines that are at least 6 cm apart. Using the parallel edges of your double-edged straightedge, make a parallelogram. Label your parallelogram *LOVE*.

Step 2 Let's look at the opposite angles. Measure the angles of parallelogram *LOVE*. Compare a pair of opposite angles using patty paper or your protractor. Compare results with your group. Copy and complete the conjecture.



Step 1

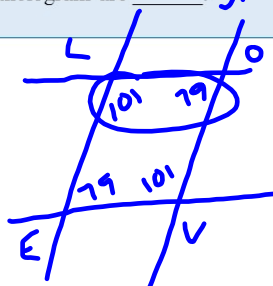


Step 2

Parallelogram Opposite Angles Conjecture

C-41

The opposite angles of a parallelogram are congruent



Two angles that share a common side in a polygon are consecutive angles. In parallelogram $LOVE$, $\angle LOV$ and $\angle EVO$ are a pair of consecutive angles. The consecutive angles of a parallelogram are also related.

Step 3 Find the sum of the measures of each pair of consecutive angles in parallelogram $LOVE$. Share your observations with your group. Copy and complete the conjecture.

Parallelogram Consecutive Angles Conjecture

C-42

The consecutive angles of a parallelogram are Supplementary

Step 4 Describe how to use the two conjectures you just made to find all the angles of a parallelogram with only one angle measure given.

Step 5 Next let's look at the opposite sides of a parallelogram. With your compass or patty paper, compare the lengths of the opposite sides of the parallelogram you made.

Share your results with your group. Copy and complete the conjecture.

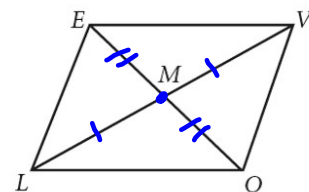
Parallelogram Opposite Sides Conjecture

C-43

The opposite sides of a parallelogram are congruent

Step 6 Finally, let's consider the diagonals of a parallelogram. Construct the diagonals \overline{LV} and \overline{EO} , as shown below. Label the point where the two diagonals intersect point M .

Step 7 Measure \overline{LM} and \overline{VM} . What can you conclude about point M ? Is this conclusion also true for diagonal \overline{EO} ? How do the diagonals relate?



Share your results with your group. Copy and complete the conjecture.

Parallelogram Diagonals Conjecture

C-44

The diagonals of a parallelogram bisect each other



INVESTIGATION SOLUTION

Parallelogram Opposite Angles Conjecture

C-41

The opposite angles of a parallelogram are **congruent**.

Step 3 Find the sum of the measures of each pair of consecutive angles in parallelogram *LOVE*.

180°

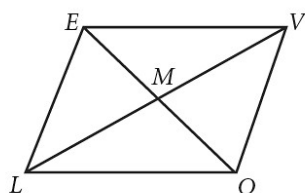
Parallelogram Consecutive Angles Conjecture

C-42

The consecutive angles of a parallelogram are **supplementary**.

Step 4 Describe how to use the two conjectures you just made to find all the angles of a parallelogram with only one angle measure given.

The opposite angle will have the same measure and the two consecutive angles will have measure 180° minus the measure of the angle.



Parallelogram Opposite Sides Conjecture

C-43

The opposite sides of a parallelogram are **congruent**.

Parallelogram Diagonals Conjecture

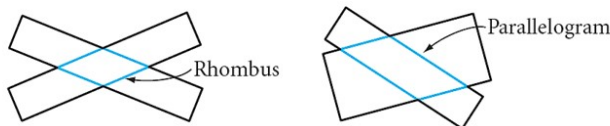
C-44

The diagonals of a parallelogram **bisect each other**.



Properties of Special Parallelograms

Recall that a **rhombus** is a parallelogram with four congruent sides, or an equilateral parallelogram. In Chapter 3, you learned how to construct a rhombus using a compass and straightedge, or using patty paper. Now you know a quicker and easier way, using a double-edged straightedge. To construct a parallelogram that is not a rhombus, you need two double-edged straightedges of different widths.



Now let's investigate some properties of rhombuses.

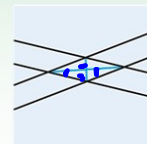
INVESTIGATION 2

Do Rhombus Diagonals Have Special Properties?

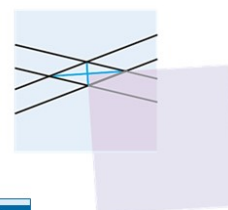
YOU WILL NEED:
patty paper,
straightedge,
protractor
(optional)

Step 1 Draw in both diagonals of the rhombus you created in Investigation 1.

Step 2 Use the corner of a patty paper or a protractor to measure the angles formed by the intersection of the two diagonals. Are the diagonals perpendicular? Compare your results with your group. Also, recall that a rhombus is a parallelogram and that the diagonals of a parallelogram bisect each other. Combine these two ideas into your next conjecture.



Step 1

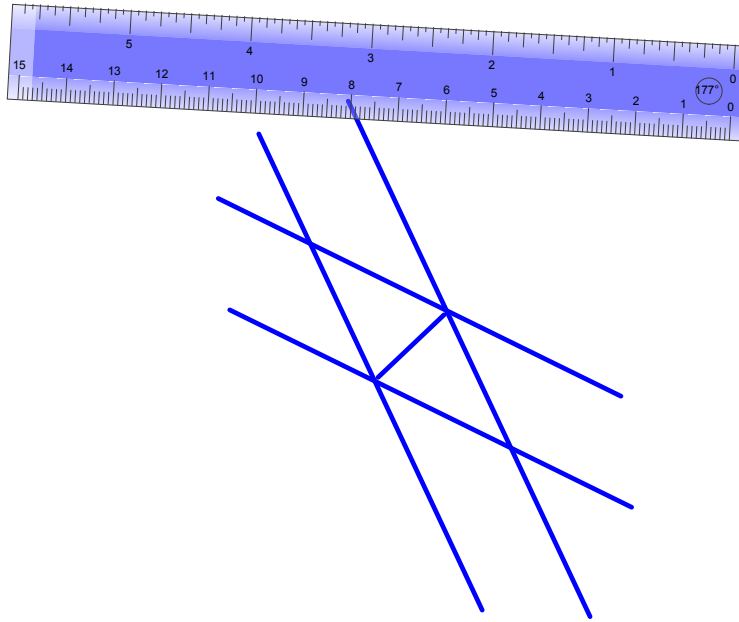


Step 2

Rhombus Diagonals Conjecture

C-46

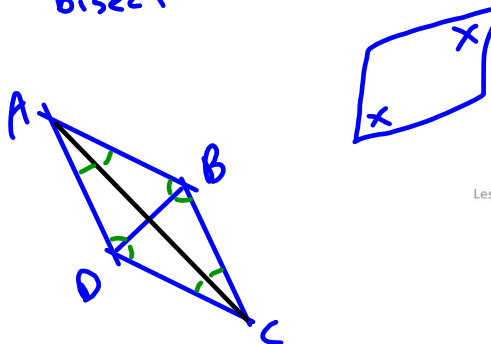
The diagonals of a rhombus are perpendicular, and they bisect each other



Step 3 The diagonals and the sides of the rhombus form two angles at each vertex. Fold your patty paper to compare each pair of angles. What do you observe? Compare your results with your group. Copy and complete the conjecture.

Rhombus Angles Conjecture C-47
 The _____ of a rhombus _____ the angles of the rhombus.

diagonals bisect





INVESTIGATION 2 SOLUTION

Rhombus Diagonals Conjecture

C-46

The diagonals of a rhombus are **perpendicular**, and they **bisect each other**.

Rhombus Angles Conjecture

C-47

The **diagonals** of a rhombus **bisect** the angles of the rhombus.

LESSON**5.5**

Properties of Special Parallelograms

So far you've made conjectures about a rhombus, a quadrilateral with four congruent sides. Now let's look at quadrilaterals with four congruent angles. A rectangle is a parallelogram with four congruent angles, or an equiangular parallelogram. What special properties do they have?



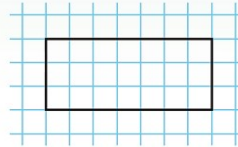

INVESTIGATION 3

YOU WILL NEED:
graph paper,
compass

Do Rectangle Diagonals Have Special Properties?

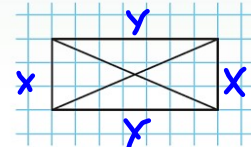
Now let's look at the diagonals of rectangles.

Step 1 Draw a large rectangle using the lines on a piece of graph paper as a guide.



Step 1

Step 2 Draw in both diagonals. With your compass, compare the lengths of the two diagonals.



Step 2

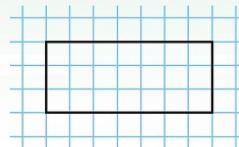
Compare results with your group. In addition, recall that a rectangle is also a parallelogram. So its diagonals also have the properties of a parallelogram's diagonals. Combine these ideas to complete the conjecture.

Rectangle Diagonals Conjecture
C-48

The diagonals of a rectangle are _____ and _____.

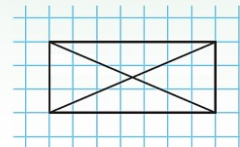

**INVESTIGATION 3
SOLUTION**

Step 1 Draw a large rectangle using the lines on a piece of graph paper as a guide.



Step 1

Step 2 Draw in both diagonals. With your compass, compare the lengths of the two diagonals.



Step 2

Compare results with your group. In addition, recall that a rectangle is also a parallelogram. So its diagonals also have the properties of a parallelogram's diagonals. Combine these ideas to complete the conjecture.

Rectangle Diagonals Conjecture
C-48

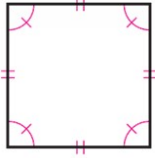
The diagonals of a rectangle are **congruent** and **bisect each other**.



INVESTIGATION 4

What Are the Properties of the Diagonals of a Square?

This final investigation is really a "thought experiment." What happens if you combine the properties of a rectangle and a rhombus? We call the shape a square. You can think of a square as a special rhombus and also a special rectangle. So you can define it in at least two different ways.



A **square** is an equiangular rhombus.

Or

A **square** is an equilateral rectangle.

A square is a parallelogram, as well as both a rectangle and a rhombus. Thus the square has the diagonal properties of all three. Discuss with your group members what you know about the diagonals of these three special parallelograms, then copy and complete this conjecture.

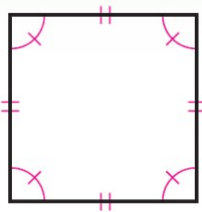
Square Diagonals Conjecture

C-49

The diagonals of a square are _____, _____ and _____.



INVESTIGATION 4 SOLUTION



Square Diagonals Conjecture

C-49

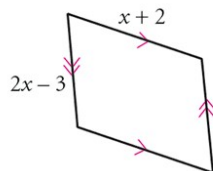
The diagonals of a square are **congruent**, **perpendicular** and **bisect each other**.



Properties of Parallelograms

Extra Example

If the perimeter of the quadrilateral equals 52 cm, what is the value of x ?

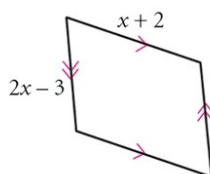


Properties of Parallelograms

Extra Example

ANSWER

$$x = 9 \text{ cm}$$

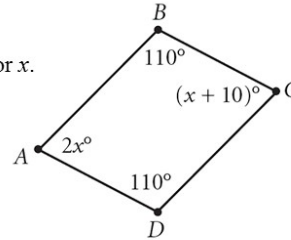




Properties of Parallelograms

Closing Question

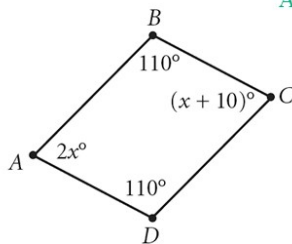
Given parallelogram $ABCD$, solve for x .



Properties of Parallelograms

Closing Question

ANSWER



There is no solution. There are several ways to look at this problem. In a parallelogram, opposite angles are congruent, so $2x = x + 10$, or $x = 10$. But, in a parallelogram, consecutive angles are supplementary angles. But, if $x = 10$, then the measure of each adjacent angle is only 130° . Since $\angle B$ and $\angle C$ are supplementary, $110^\circ + (x + 10)^\circ = 180^\circ$, or $x = 60$. But $\angle A$ and $\angle D$ are also supplementary, $110^\circ + (2x)^\circ = 180^\circ$, or $x = 35$.

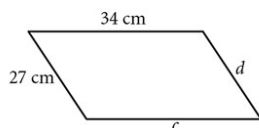
5.4 Exercises

pages 284 – 287

Use your new conjectures in the following exercises.

In Exercises 1–6, each figure is a parallelogram.

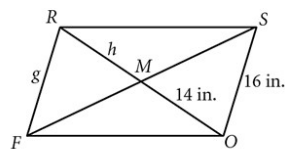
1. $c = ?$
 $d = ?$



2. $a = ?$
 $b = ?$



3. $g = ?$
 $h = ?$



Assignment

From the Textbook: $\left\{ \begin{array}{l} \text{pp. 284-286 1-6, 13} \\ \text{pp. 292-293 1-16} \end{array} \right.$

Also: Parallelograms Worksheet