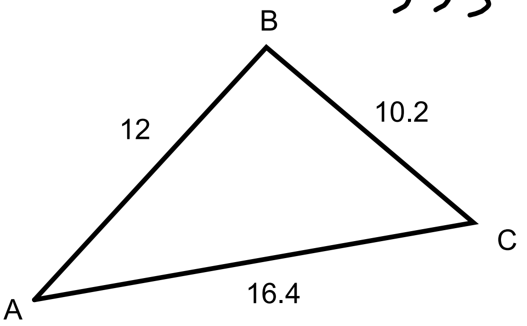
Warm-Up

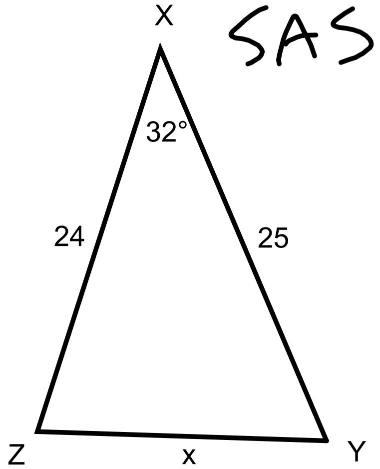
Apply the Law of Sines to the triangles shown. What do you notice?

1. SSS



$$\frac{10.2}{\sin A} = \frac{12}{\sin C} = \frac{16.4}{\sin B}$$

2. SAS



$$\frac{24}{\sin Y} = \frac{x}{\sin 32}$$

LESSON 12.4

The Law of Cosines

Objective

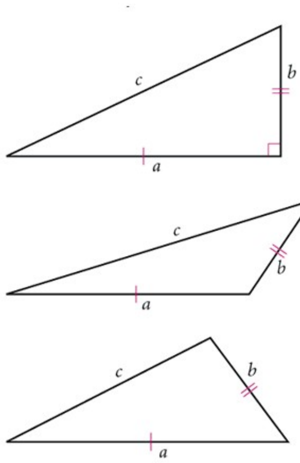
A. I can use the Law of Cosines to find missing sides and angles of triangles

LESSON 12.4

The Law of Cosines

Launch

For each triangle write an equality or inequality that describes the relationship between c^2 and $a^2 + b^2$.



$a^2+b^2=c^2$

$a^2+b^2<c^2$

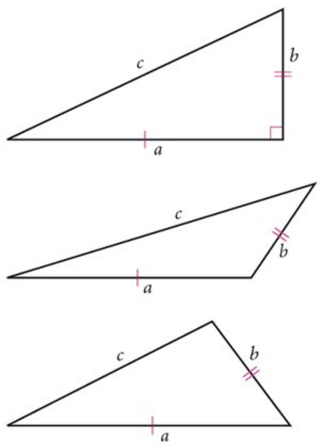
$a^2+b^2>c^2$

LESSON 12.4

The Law of Cosines

Launch

For each triangle write an equality or inequality that describes the relationship between c^2 and $a^2 + b^2$.



Right triangle:
$c^2 = a^2 + b^2$

Obtuse triangle:
$c^2 > a^2 + b^2$

Acute triangle:
$c^2 < a^2 + b^2$

LESSON 12.4

The Law of Cosines

You've solved a variety of problems with the Pythagorean Theorem. It is perhaps your most important geometry conjecture. In Chapter 10, you found that the distance formula is really just the Pythagorean Theorem.

The Pythagorean Theorem is very powerful, but its use is still limited to right triangles. Recall from Chapter 10 that the Pythagorean Theorem does not work for acute triangles or obtuse triangles. You might ask, "What happens to the Pythagorean equation for acute triangles or obtuse triangles?"

LESSON 12.4

The Law of Cosines

If the legs of a right triangle are brought closer together so that the right angle becomes an acute angle, you'll find that $c^2 < a^2 + b^2$. In order to make this inequality into an equality, you would have to subtract something from $a^2 + b^2$.

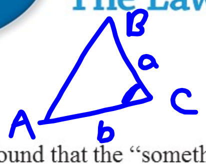
$c^2 = a^2 + b^2 - \textit{something}$

If the legs are widened to form an obtuse angle, you'll find that $c^2 > a^2 + b^2$. Here, you'd have to add something to make an equality.

$c^2 = a^2 + b^2 + \textit{something}$

LESSON 12.4

The Law of Cosines

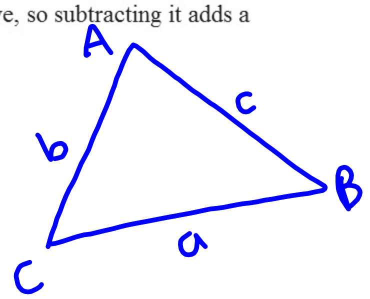


Mathematicians found that the "something" was $\boxed{2ab \cos C}$. The Pythagorean Theorem generalizes to all triangles with a property called the **Law of Cosines**.

Law of Cosines C-99

For any triangle with sides of lengths a, b, and c, and with C the angle opposite the side with length c, $c^2 = a^2 + b^2 - 2ab \cos C$.

For obtuse angles, the expression $2ab \cos C$ is negative, so subtracting it adds a positive quantity.



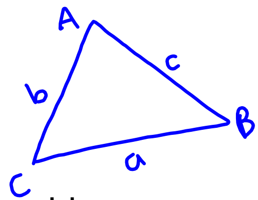
History CONNECTION

Many, if not most, geometric discoveries were made inductively and then proved deductively. Occasionally geometric properties are discovered deductively. That is surely how the law of sines and the law of cosines were discovered!

Lesson 12.4 The Law of Cosines

The Law of Cosines

$$\boxed{c^2 = a^2 + b^2 - \underline{2ab \cos C}}$$



C is the angle opposite side c, and a and b are the other two sides about angle C. The following equations also work, but are essentially the same as the equation above.

Also

90

$$\boxed{a^2 = b^2 + c^2 - 2bc \cos A}$$

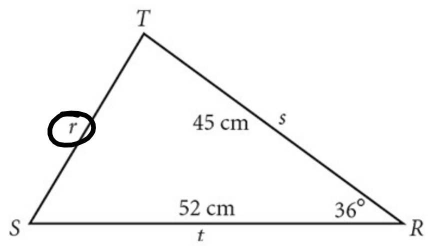
or

$$\boxed{b^2 = a^2 + c^2 - 2ac \cos B}$$

Remember, use the Law of Cosines when you are given either 3 sides of a triangle, or two sides and the included angle. Otherwise, use the Law of Sines to find the missing information.

EXAMPLE A

Find the length of side $\overline{ST}$ in triangle *SRT*.



$r^2 = 45^2 + 52^2 - 2 \cdot 45 \cdot 52 \cos 36$

$r = \sqrt{45^2 + 52^2 - 2 \cdot 45 \cdot 52 \cos 36}$

optional

$* r = \sqrt{4729 - 4680 \cos 36}$

$r \approx 30.7$

SOLUTION

To find r, use the Law of Cosines:

$c^2 = a^2 + b^2 - 2ab\cos C$ — The Law of Cosines.

Using the variables in this problem, the Law of Cosines becomes

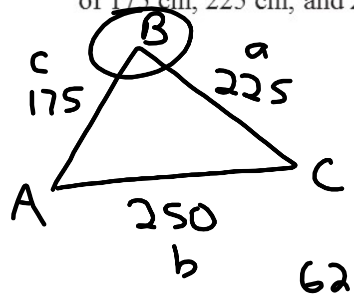
$r^2 = s^2 + t^2 - 2st\cos R$	Substitute r for c, s for a, t for b, and R for C.
$r^2 = 45^2 + 52^2 - 2(45)(52)(\cos 36°)$	Substitute 45 for s, 52 for t, and 36° for R.
$r = \sqrt{45^2 + 52^2 - 2(45)(52)(\cos 36°)}$	Take the positive square root of both sides.
$r \approx 31$	Evaluate.

The length of side $\overline{ST}$ is about 31 cm.

EXAMPLE B

SSS

What is the measure of the largest angle of a triangle with sides of 175 cm, 225 cm, and 250 cm?



$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$250^2 = 225^2 + 175^2 \underbrace{-2\cdot 225\cdot 175}\cos B$$

$$62500 = 50625 + 30625 - 78750\cos B$$

$$\frac{-18750}{-78750} = \frac{-78750\cos B}{-78750}$$

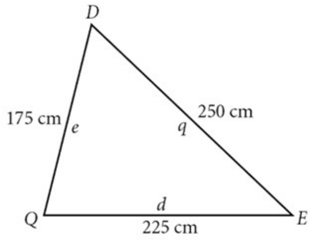
$$\frac{-18750}{-78750} = \cos B$$

$$\cos^{-1}\left(\frac{-18750}{-78750}\right) = B$$

$$76.2° \approx B$$

SOLUTION

First we draw and label a triangle with the given information. In $\triangle QED$, $\angle Q$ is the largest angle because it is opposite the largest side. Use the Law of Cosines and solve for Q.



$q^2 = e^2 + d^2 - 2ed\cos Q$	The Law of Cosines with respect to $\angle Q$.
$\cos Q = \dfrac{q^2 - e^2 - d^2}{-2ed}$	Solve for cos Q.
$\cos Q = \dfrac{250^2 - 175^2 - 225^2}{-2(175)(225)}$	Substitute known values.
$Q = \cos^{-1}\left[\dfrac{250^2 - 175^2 - 225^2}{-2(175)(225)}\right]$	Take the inverse cosine of both sides.
$Q \approx 76$	Evaluate.

The measure of $\angle Q$ is about 76°.

LESSON 12.4

The Law of Cosines

Summarize

LESSON 12.4

The Law of Cosines

Summarize

- What information do you need about a triangle in order to use the Law of Cosines?
- What happens to the Law of Cosines when the included angle measure is 90˚?

LESSON 12.4

The Law of Cosines

Summarize

- What information do you need about a triangle in order to use the Law of Cosines?

  You need three sides, or two sides and an angle. If you know the lengths of the three sides, you can use the Law of Cosines to solve for one angle and find the other parts of the triangle using the Law of Sines.

- What happens to the Law of Cosines when the included angle measure is 90˚?

  The cosine is 0, so the equation becomes the Pythagorean Theorem.

LESSON 12.4

The Law of Cosines

Extra Example

What is the measure of the largest angle in a triangle with side lengths equal to 5 in., 7 in., and 10 in.?