Warm Up

1. If $\tan A = \frac{15}{8}$

Find cos A

2. If $\cos A = \frac{24}{25}$

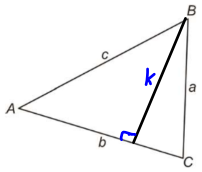
Find sin A

The Law of Sines

NAME ____________________

Right triangle trigonometry can be used to solve problems involving right triangles. However, many interesting problems involve non-right triangles. In this lesson, you will use right triangle trigonometry to develop the *Law of Sines*. The law of sines is important because it can be used to solve problems involving non-right triangles as well as right triangles.

Consider oblique ΔABC shown to the right.



1. Sketch an altitude from vertex B.
2. Label the altitude *k*.
3. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles, and $\angle C$ is contained in the other. Using right triangle trigonometry, write two equations, one involving sin *A*, and one involving sin *C*.

$\sin A = \frac{k}{c}$ $\sin C = \frac{k}{a}$

4. Notice that each of the equations in Question 3 involves *k*. (Why does this happen?) Solve each equation for *k*.

$c \cdot \sin A = k$ $a \cdot \sin C = k$

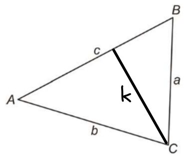
5. Since both equations in Question 4 are equal to *k*, they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.

$c \sin A = a \sin C$

6. Notice that the equation in Question 5 no longer involves *k*. (Why not?) Write an equation equivalent to the equation in Question 5, regrouping *a* with sin *A* and *c* with sin *C*.

$\frac{\sin A}{a} = \frac{\sin C}{c}$ or $\frac{a}{\sin A} = \frac{c}{\sin C}$

Again, consider oblique ΔABC.



7. This time, sketch an altitude from vertex C.

8. Label the altitude *k*.

9. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles and $\angle B$ is contained in the other. Using right triangle trigonometry, write two equations, one involving $\sin A$ and one involving $\sin B$.

$\sin A = \frac{k}{b}$ $\qquad$ $\sin B = \frac{k}{a}$

10. Notice that each of the equations in Question 9 involves *k*. (Why does this happen?) Solve each equation for *k*.

$b \sin A = k \qquad a \sin B = k$

11. Since both equations in Question 10 are equal to *k*, they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.

$b \sin A = a \sin B$

12. Notice that the equation in Question 11 no longer involves *k*. (Why not?) Write an equation equivalent to the equation in Question 11, regrouping *a* with $\sin A$ and *b* with $\sin B$.

$\frac{\sin A}{a} = \frac{\sin B}{b}$ or $\frac{a}{\sin A} = \frac{b}{\sin B}$

13. Use the equations in Question 6 and Question 12 to write a third equation involving *b*, *c*, $\sin B$, and $\sin C$.

$\frac{\sin B}{b} = \frac{\sin C}{c}$ or $\frac{b}{\sin B} = \frac{c}{\sin C}$

Together, the equations in Questions 6, 12, and 13 form the *Law of Sines*. The law of sines is important, because it can be used to solve problems involving both right and non-right triangles, because it involves only the sides and angles of a triangle.

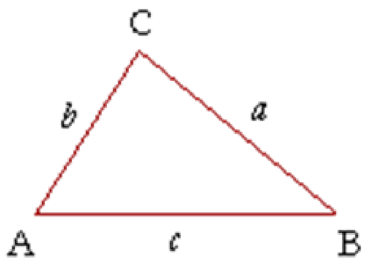
Section 12.3
Law of Sines

So far, we have only been dealing with right triangles (triangles with a $90°$ angle).

If triangles are not right triangles, then we can use the *LAW OF SINES* to find missing parts of the triangles.

LAW OF SINES

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

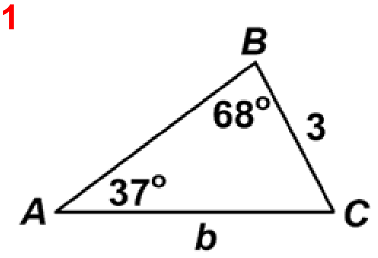


or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Find the missing side or angle for each triangle.
Round to the nearest hundredths place if necessary.

EX 1



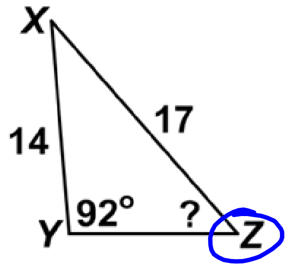
$$\frac{3}{\sin 37} = \frac{b}{\sin 68}$$

$$3\sin 68 = b\sin 37$$

$$\frac{3\sin 68}{\sin 37} = b$$

$$4.62 \approx b$$

EX 2



$$14 \cdot \frac{\sin Z}{14} = \frac{\sin 92}{17} \cdot 14$$

$$\sin Z = \frac{14\sin 92}{17}$$

$$Z = \sin^{-1}\frac{14\sin 92}{17}$$

$$Z \approx 55.39$$

Law of Sines Practice

Workbook Pg. 90 4-10

From Friday:

- Textbook p. 591-593: 10-16