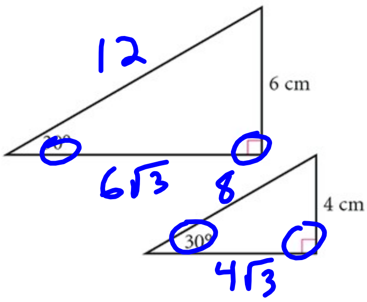
Warm-Up

Find the missing sides. Describe your method.

Are these triangles similar? Explain how you know.



LESSON 12.1

Trigonometric Ratios

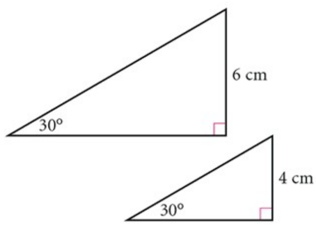
Warm-Up

Find the missing sides. Describe your method.

12 cm, $6\sqrt{3}$ cm, 8 cm, $4\sqrt{3}$ cm, Using the side ratios of a 30°-60°-90° triangle.

Are these triangles similar? Explain how you know.

Yes, they are similar because of the AA Triangle Similarity conjecture.



12.1 Trigonometric Ratios Day 1

A. I can define the sine, cosine, and tangent ratios

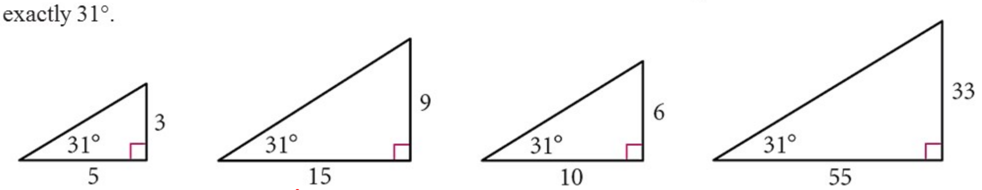
B. I can explain and use the relationship between the sine and cosine of complementary angles

LESSON 12.1

Trigonometric Ratios

Trigonometry is the study of the relationships between the sides and the angles of triangles. In this lesson you will discover some of these relationships for right triangles.

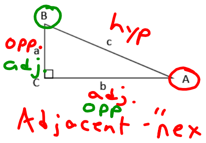
When studying right triangles, early mathematicians discovered that whenever the ratio of the shorter leg's length to the longer leg's length was close to a specific fraction, the angle opposite the shorter leg was close to a specific measure. They found this (and its converse) to be true for all similar right triangles. For example, in every right triangle in which the ratio of the shorter leg's length to the longer leg's length is $\frac{3}{5}$, the angle opposite the shorter leg is almost exactly 31°.



$\frac{15}{9} = \frac{5}{3}$ $\frac{10}{6} = \frac{5}{3}$ $\frac{55}{33} = \frac{5}{3}$

Opposite/Adjacent/Hypotenuse

To understand sine, cosine, and tangent, you must label the sides of a right triangle in relation to the specific angle in which you are looking. Use the triangle below to answer the following questions.



Adjacent - "next to"

Which side is the hypotenuse? $\overline{AB}$

Which side is opposite of $\angle A$? $\overline{BC}$

Which side is adjacent to $\angle A$? $\overline{AC}$

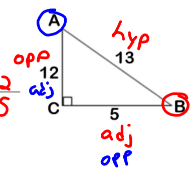
Which side is opposite of $\angle B$? $\overline{AC}$

Which side is adjacent to $\angle B$? $\overline{BC}$

Sine (sin), Cosine (cos), and Tangent (tan)

The **sine** of an angle is the ratio comparing the **side opposite the angle to the hypotenuse**. The **cosine** of an angle is the ratio comparing the **side adjacent to the angle to the hypotenuse**. The **tangent** of an angle is the ratio comparing the **side opposite the angle to the side adjacent to the angle**.

For example, $\sin B = \frac{12}{13}\left(\frac{\textit{side opposite of angle } B}{\textit{hypotenuse of } \Delta ABC}\right)$ and $\cos B = \frac{5}{13}\left(\frac{\textit{side adjacent to angle } B}{\textit{hypotenuse of } \Delta ABC}\right)$



Find the other 4 trigonometric ratios of ΔABC.

1. $\cos A = \frac{12}{13}$ 2. $\sin A = \frac{5}{13}$ 3. $\tan A = \frac{5}{12}$ 4. $\tan B = \frac{12}{5}$

$\cos = \frac{a}{h}$ $\sin = \frac{o}{h}$ $\tan = \frac{o}{a}$

To remember the trigonometric ratios, we can use the following mnemonic:

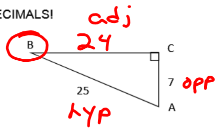
SOH-CAH-TOA

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

Theta

SOH CAH TOA

Examples: Use the triangle below to find sin, cos, tan. NO DECIMALS!



5. sin A = $\frac{24}{25}$
6. cos A = $\frac{7}{25}$
7. tan A = $\frac{24}{7}$
8. sin B = $\frac{7}{25}$
9. cos B = $\frac{24}{25}$
10. tan B = $\frac{7}{24}$

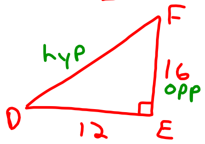
$7^2 + x^2 = 25^2$

$49 + x^2 = 625$

$x^2 = 576$

$x = 24$

11. In ΔDEF, $\angle E$ is the right angle. Suppose $\boxed{\tan F = \frac{12}{16}}$. Find $\sin D$.



$\tan F = \frac{opp}{adj}$

$12^2 + 16^2 = x^2$

$400 = x^2$

$20 = x$

$\sin D = \frac{opp}{hyp}$

$\sin D = \frac{16}{20} = \boxed{\frac{4}{5}}$

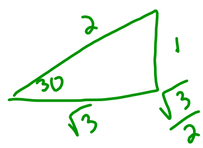
The main reason we use sine, cosine, and tangent is to find missing parts of right triangles when the Pythagorean Theorem, geometric mean, or special right triangle relationships will not work.

Finding Missing Sides

You can find trigonometric ratios using your calculator! Each angle has a specific ratio programmed for it for sine, cosine, and tangent. These are always the same because all right triangles with congruent corresponding angles are similar; thus the sides will always form the same ratios!

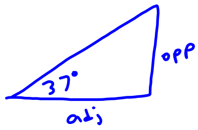
****** Make sure your calculator is in** degree mode ****

rad



Examples: Find the values using your calculator. Round to the nearest ten-thousandths.

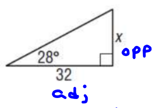
12. $\sin 45° =$ .7071   13. $\cos 87° =$ .0523   14. $\tan 37° =$ .7536



Now we will set up equations using sine, cosine, and tangent in order to find missing sides of right triangles.

Examples: Find the missing side lengths.

15. TOA

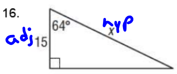


$32 \cdot \tan 28 = \frac{x}{32} \cdot 32$

$32 \tan 28 = x$

$17.01 \approx x$

16. CAH



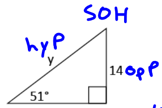
$x \cdot \cos 64 = \frac{15}{x} \cdot x$

$\frac{x \cdot \cos 64}{\cos 64} = \frac{15}{\cos 64}$

$x = \frac{15}{\cos 64}$

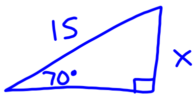
$x \approx 34.22$

17. SOH

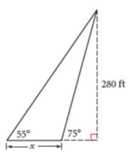


$\sin 51 = \frac{14}{y}$

18. A 15-foot ladder leans against a wall. The **angle of elevation** (the angle between the ladder and ground) is 70°. How far up the wall does the ladder reach?



19. Find the value of x.



LESSON 12.1

Trigonometric Ratios

Summarize

- Why is the sine of an angle the same as the cosine of its complement?
- How does using trigonometric ratios compare with using the Pythagorean Theorem or similar triangles to find lengths and distances?
- Why doesn't the scale factor depend on the angle?

LESSON 12.1

Trigonometric Ratios

Summarize

- Why is the sine of an angle the same as the cosine of its complement?

  The sine ratio is $\frac{length\ of\ the\ opposite\ leg}{length\ of\ thr\ hypotenuse.}$. The cosine ratio is $\frac{length\ of\ the\ adjacent\ leg}{length\ of\ the\ hypotenuse.}$. For the acute angles of a right triangle, the side opposite one vertex is adjacent to the other vertex.

- How does using trigonometric ratios compare with using the Pythagorean Theorem or similar triangles to find lengths and distances?

  To use the Pythagorean Theorem, you need to know the lengths of two sides of a right triangle. To use similar triangles, you need two triangles. To use trigonometric ratios, you need only one triangle, but it needs to be a right triangle and you need to know the length of one side and the measure of one nonright angle.

- Why doesn't the scale factor depend on the angle?

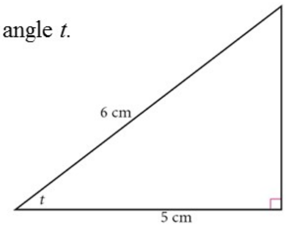
  By AA, all right triangles with a given nonright angle are similar, so they all have the same ratios of corresponding sides.

LESSON 12.1

Trigonometric Ratios

Extra Example

Find the measure of angle t.



LESSON 12.1

Trigonometric Ratios

Extra Example

ANSWER

Find the measure of angle t.

$t = 33.6°$