LESSON 11.8

# Similarity and Volume

## Objectives

11.8 Similarity and Volume

A. I can use the relationship between volumes of solids to determine similarity



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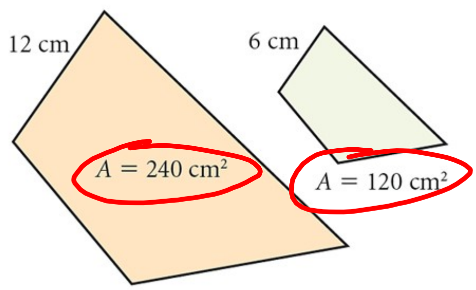
# Similarity and Volume

## Warm-Up

Are these two quadrilaterals similar? Explain why or why not.




2 : 1

4 : 1

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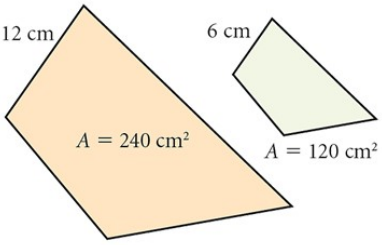
# Similarity and Volume

scale factor$^2$ = ratio of Areas

Are these two quadrilaterals similar? Explain why or why not.




No because the ratio of the sides is 2:1 and the ratio of the areas is also 2:1. If they were similar, the ratio of the areas would be 4:1.



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# Similarity and Volume

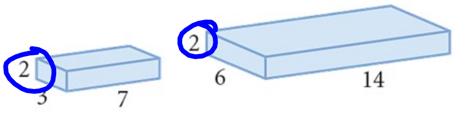
The statues at right are giant versions of an Oscar, the Academy Awards statuette that is handed out each year for excellence in the motion-picture industry. Assume that a statue is similar to the actual Oscar, but six times as long in each dimension. If it costs \$250 to gold-plate the surface of the actual Oscar, how much would it cost to gold-plate the statue? As you learned in Chapter 8, the surface area is not six times greater, but $6^2$, or 36 times greater, so the cost would be \$9,000!

How are their weights related? The actual Oscar weighs 8.5 pounds. If the statue were made of the same material as the actual Oscar, its weight would not be six times greater, or even 36 times greater. It would weigh 1836 pounds! In this lesson you will discover why.

Solids that have the same shape but not necessarily the same size are similar. All cubes are similar, but not all prisms are similar. All spheres are similar, but not all cylinders are similar. Two polyhedrons are similar if all their corresponding faces are similar and the lengths of their corresponding edges are proportional. Two right cylinders (or right cones) are similar if their radii and heights are proportional.

### EXAMPLE A

Are these right rectangular prisms similar?



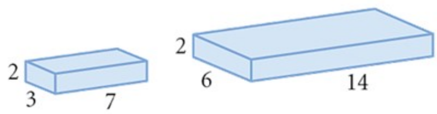

$$\frac{3}{6} = \frac{7}{14} \neq \frac{2}{2}$$



### SOLUTION

The two prisms are not similar because the corresponding edges are not proportional.

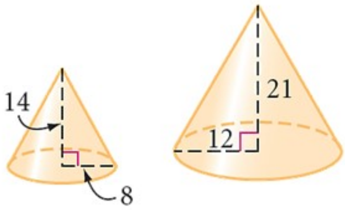
$$\frac{2}{2} \neq \frac{3}{6} = \frac{7}{14}$$



### EXAMPLE B

Are these right circular cones similar?

$$\frac{14}{21} = \frac{2}{3}$$



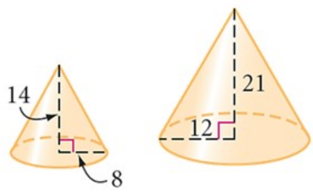

$$\frac{8}{12} = \frac{2}{3}$$



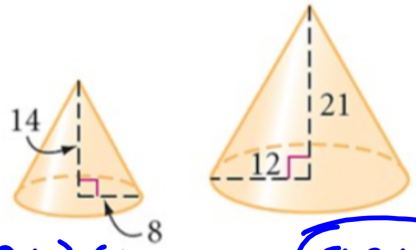
### SOLUTION

The two cones are similar because the radii and heights are proportional.

$$\frac{8}{12} = \frac{14}{21}$$



Find the volume of each cone. How do the values compare?




$$\frac{\frac{896\pi}{3}}{1008\pi}$$

$$V_{small} = \frac{\pi(8)^2(14)}{3} = \frac{896\pi}{3}$$

Ratio of Volumes
8 : 27
$2^3 : 3^3$

$$V_{large} = \frac{\pi(12)^2(21)}{3} = 1008\pi$$

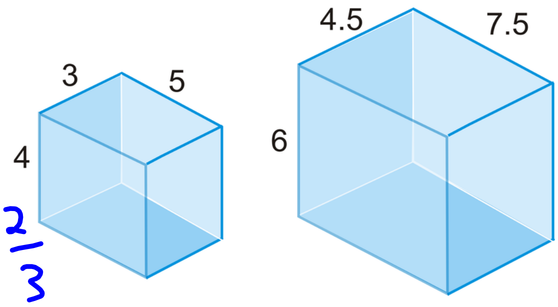
## EXAMPLE C

The two prisms below are similar. What is their scale factor?

$\frac{4}{6} = \frac{2}{3}$

$\frac{3}{4.5} = \frac{2}{3}$

$\frac{5 \times 10}{7.5 \times 10} = \frac{50}{75} = \frac{2}{3}$




S.F. = 2 : 3

ratio of volumes = 8 : 27

Find the volume of each prism. How do their volumes compare?

$V_{small} = 60$ $V_{large} = 202.5$

$$\frac{60}{202.5} = \frac{8}{27}$$

In your groups, make a conjecture about how the volumes of similar figures are related.

**Proportional Volumes Conjecture** C-96

If corresponding edge lengths (or radii, or heights) of two similar solids compare in the ratio $\frac{m}{n}$, then their volumes compare in the ratio $m^3/n^3$

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

**Proportional Volumes Conjecture** C-96

If corresponding edge lengths (or radii, or heights) of two similar solids compare in the ratio $\frac{m}{n}$, then their volumes compare in the ratio $\frac{m^3}{n^3}$ or $\left(\frac{m}{n}\right)^3$.

The reason behind the Proportional Volumes Conjecture is that volume is a three-dimensional measure. Because volume is related to the product of a two-dimensional base and height, when all three dimensions are doubled, the volume increases by a factor of eight.

Let's look at an application of both the Proportional Areas Conjecture and Proportional Volumes Conjecture.

### EXAMPLE D

The diameter of a soccer ball is about 8.75 in. and the diameter of a tennis ball is about 2.5 in. How many times more surface material is needed to make the outside of a soccer ball than a tennis ball? How many times more air does a soccer ball hold than a tennis ball?

Scale factor $= \frac{8.75}{2.5} = \frac{7}{2}$

square S.F. Ratio of surface area $= \frac{49}{4} \rightarrow$ 12.25 times the material




cube S.F. Ratio of air $= \frac{343}{8} \rightarrow$ 42.875 times the air.

### SOLUTION

The ratio of the diameters is $\frac{8.75}{2.5}$, or $\frac{7}{2}$, so the ratio of the surface areas is $\left(\frac{7}{2}\right)^2$, or $\frac{49}{4}$, and the ratio of the volumes is $\left(\frac{7}{2}\right)^3$, or $\frac{343}{8}$. Therefore, the soccer ball requires $12\frac{1}{4}$ times as much material to make and holds $42\frac{7}{8}$ times as much air.

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## Extra Example

$4^3 : 3^3$

$x^3$ third The ratio of the volumes of two similar cubes is 64:27.

$x^2$ second What is the ratio of the areas of the bases? 16 : 9

$x^1$ first What is the ratio of the sides? 4 : 3





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# Similarity and Volume

## Extra Example

ANSWER

The ratio of the volumes of two similar cubes is 64:27.

What is the ratio of the areas of the bases? 16:9

What is the ratio of the sides? 4:3

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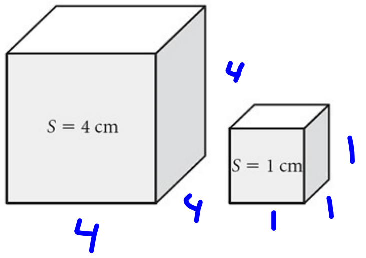
# Similarity and Volume

## Closing Question

Are the cubes below similar? If so, what is the ratio of their surface areas and the ratio of their volumes?




S.F. = 4:1

Ratio of SA = 16:1

Ratio of V = 64:1



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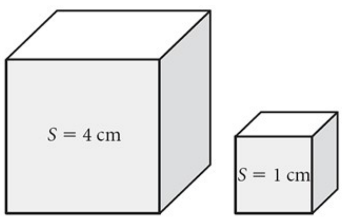
Closing Question

ANSWER




Cubes are regular solids, so they are similar. The scale factor is 4: 1, the ratio of the surface areas is 16:1, and the ratio of their volumes is 64:1.

# Assignment: Workbook page 87