



Volume of Pyramids and Cones

COMMON CORE STATE STANDARDS

Applied	Developed	Introduced
8.G.9	G.MG.1 G.GMD.1 G.GMD.3	

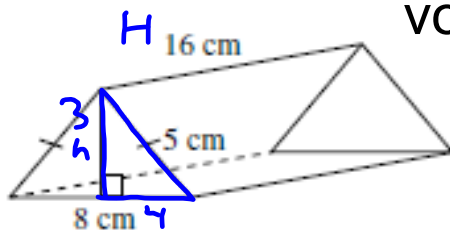
8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

G.MG.1 Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

G.GMD.3 Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.*

Warm-up:



Find the surface area and volume of the following solid:

$$B = 12 \quad V = BH$$

$$P = 18 \quad SA = LA + 2B$$

$$H = 16$$

$$V = 12 \cdot 16$$

$$SA = \underline{312 \text{ cm}^2}$$

$$V = \underline{192 \text{ cm}^3}$$

$$SA = PH + 2B$$

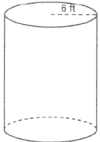
$$SA = 18(16) + 2(12)$$

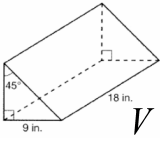
What questions do you have?

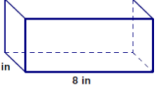
11.2 Volume of Prisms and Cylinders Practice
Geometry 3413

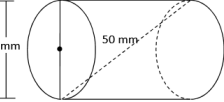
Name _____
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
Find the volume of each prism or cylinder.

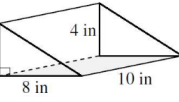
1.  $V = 540\pi ft^3$

2.  $V = 729in^3$

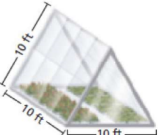
3.  $V = 240in^3$

4.  $V = 2352\pi mm^3$

5.  $V = 1050\sqrt{3}ft^3$

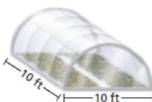
6.  $V = 160in^3$

7. **Landscaping** Ingrid is building a shelter to protect her plants from freezing. She is planning to stretch plastic sheeting over the top and the ends of a frame. Which of the frames shown will require more plastic?



$SA = 200 + 50\sqrt{3}ft^2$

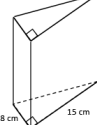
$V = 250\sqrt{3}ft^3$



$SA = 75\pi ft^2$

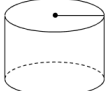
$V = 125\pi ft^3$

8. The triangular prism has a volume of 1440 cm^3 . What is the prism's height?



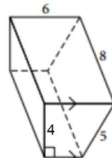
$H = 24cm$

9. The volume of the cylinder is $5175\pi\text{ mm}^3$. What is the radius of the cylinder's base?



$r = 15mm$

10. Find the volume of the trapezoidal prism below.



$V = 144units^3$

Demonstration for Volume of a Pyramid

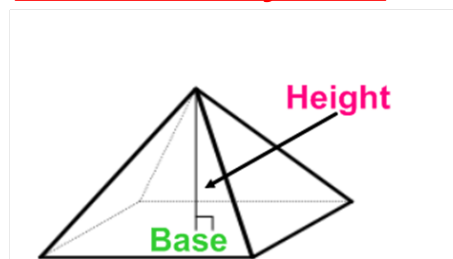
<http://www.youtube.com/watch?v=BjbilpBaA-U>

Section 11.3
Volume of Pyramids and Cones

Goals: Find the volume of pyramids and cones.

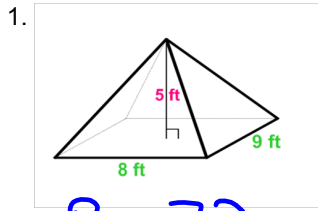
$$V = BH$$
$$V = \frac{1}{3}BH$$

Volume of a Pyramid



VOLUME OF A PYRAMID = $\frac{(\text{Area of Base})(\text{Height})}{3}$ or $\frac{BH}{3}$ or $\frac{1}{3}BH$

Examples Find the volume of the pyramid.

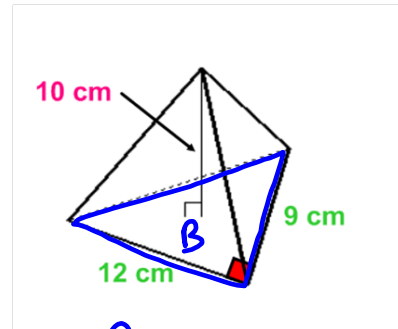


$$B = 72$$

$$H = 5$$

$$V = \frac{72 \cdot 5}{3}$$

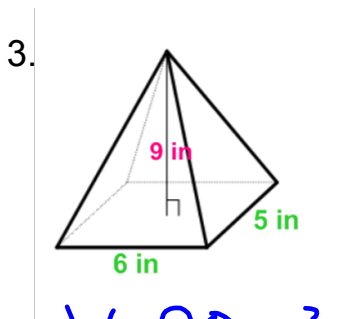
$$V = \boxed{120 \text{ ft}^3}$$



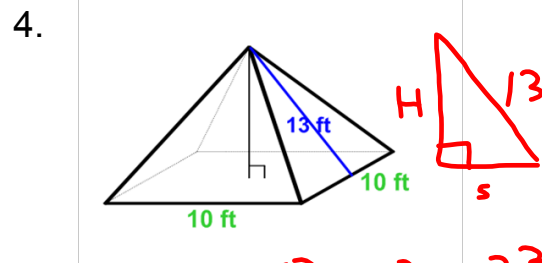
$$B = 54$$

$$H = 10$$

$$V = \frac{54 \cdot 10}{3} = \boxed{180 \text{ cm}^3}$$



$$V = 90 \text{ in}^3$$



$$5^2 + H^2 = 13^2$$

$$25 + H^2 = 169$$

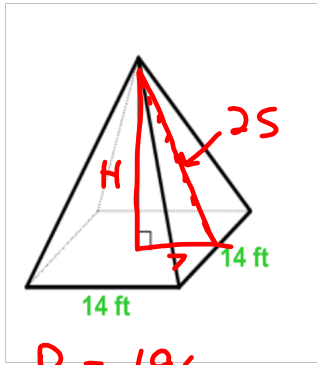
$$H^2 = 144$$

$$H = 12$$

$$B = 100$$

$$V = \frac{100 \cdot 12}{3} = \boxed{400 \text{ ft}^3}$$

5. Given a pyramid with a surface area of 896 ft^2 find the slant height, height, and volume.



$$\left[\begin{array}{l} B = 196 \\ P = 56 \\ H = 24 \end{array} \right.$$

$$SA = LA + B$$

$$896 = LA + 196$$

$$700 = LA$$

$$LA = \frac{Pl}{2}$$

$$700 = \frac{56l}{2}$$

$$1400 = 56l$$

$$25 = l$$

$$7^2 + H^2 = 25^2$$

$$49 + H^2 = 625$$

$$H^2 = 576$$

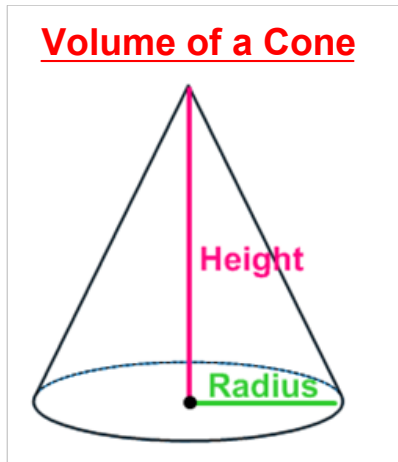
$$H = 24$$

$$V = \frac{196 \cdot 24}{3} = \boxed{1568 \text{ ft}^3}$$

Demonstration for Volume of a Cone

http://www.youtube.com/watch?v=QnVr_x7c79w

$$\frac{\pi r^2}{3} H$$

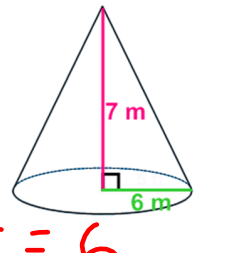


Volume of a Cone = $\frac{\pi(\text{radius})^2(\text{height})}{3}$

$$\rightarrow \frac{\pi r^2 H}{3} \text{ or } \frac{1}{3} \pi r^2 H$$

Examples: Find the volume of the cone.

6.



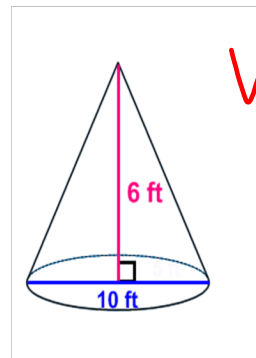
$$r = 6$$

$$H = 7$$

$$V = \frac{\pi(6)^2(7)}{3}$$

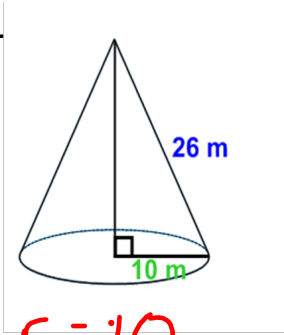
$$V = \boxed{84\pi \text{ m}^3}$$

7.



$$V = 50\pi \text{ ft}^3$$

8.



$r = 10$

$H = 24$

$$V = \frac{\pi(10)^2(24)}{3}$$

$$V = 800\pi \text{ m}^3$$

9. Find the volume of a cone with a slant height of 17 feet and a diameter of 16 feet.

$r = 8$
 $H = 15$

$$V = \frac{\pi(8)^2(15)}{3}$$

$$V = 320\pi \text{ ft}^3$$

10. The volume of a cone that is $256\pi \text{ cm}^3$ has a radius of 8 cm. Find the height, slant height, and the surface area of the cone.

$$V = \frac{\pi r^2 H}{3}$$

$$256\pi = \frac{\pi(8)^2 H}{3}$$

$$768\pi = 64\pi \cdot H$$

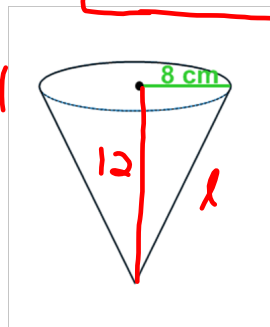
$$12 = H$$

$$8^2 + 12^2 = l^2$$

$$208 = l^2$$

$$\sqrt{208} = l$$

$$4\sqrt{13} = l$$

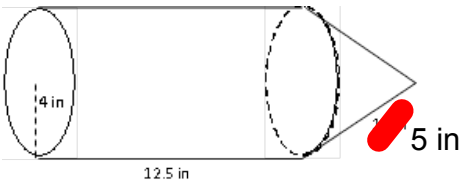


$$SA = \pi r l + \pi r^2$$

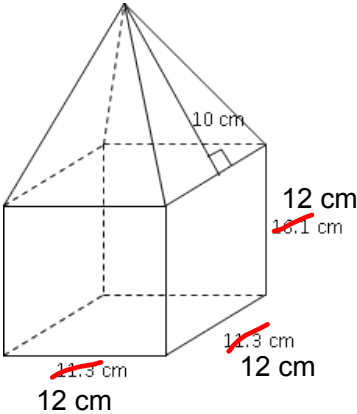
$$SA = \pi(8)(4\sqrt{13}) + \pi(8)^2$$

$$SA = 32\pi\sqrt{13} + 64\pi \text{ cm}^2$$

11. Find the total volume of the combined shape below.



12. Find the total volume of the combined shape below.





Homework:

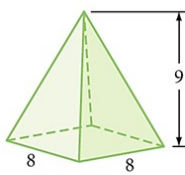
Textbook p. 550-551: 1-6, 15

11.3 Exercises

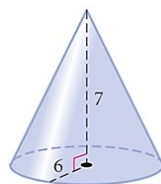
pages 550 – 552

Find the volume of each solid named in Exercises 1–6. All measurements are in centimeters.

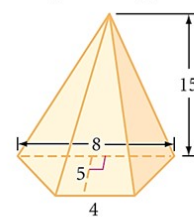
1. Square pyramid



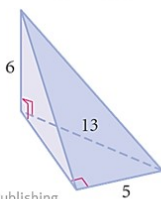
2. Cone



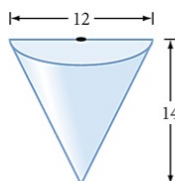
3. Trapezoidal pyramid (h)



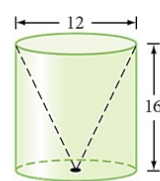
4. Triangular pyramid



5. Semicircular cone



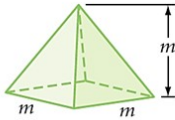
6. Cylinder with cone removed (h)



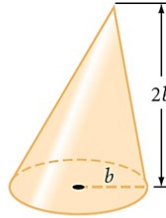
11.3 Exercises
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In Exercises 7–9, express the total volume of each solid. In Exercise 9, what percentage of the volume is filled with the liquid? All measurements are in centimeters.

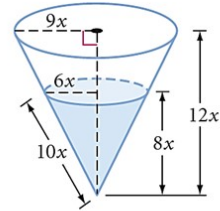
7. Square pyramid



8. Cone



9. Cone



11.3 Exercises
pages 550 – 552

10. Use the information about the base and height of each solid to find the volume. All measurements are given in centimeters.

Information about base of solid	Height of solid	Triangular pyramid	Rectangular pyramid	Trapezoidal pyramid	Cone
$b = 6, b_2 = 7, h = 6, r = 3$	$H = 20$	a. $V = \frac{1}{3}bh$	d. $V =$	g. $V =$	j. $V =$
$b = 9, b_2 = 22, h = 8, r = 6$	$H = 20$	b. $V =$	e. $V =$	h. $V =$	k. $V =$
$b = 13, b_2 = 29, h = 17, r = 8$	$H = 24$	c. $V =$	f. $V =$	i. $V =$	l. $V =$

11.3 Exercises
pages 550 – 552

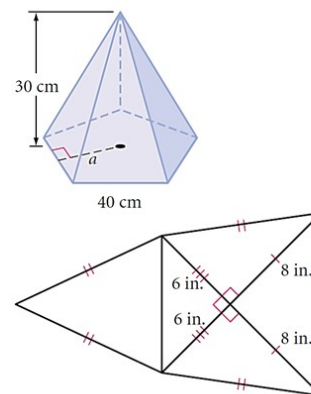
11. Sketch and label a square pyramid with height H feet and each side of the base M feet. The altitude meets the square base at the intersection of the two diagonals. Find the volume in terms of H and M .
12. Sketch and label two different circular cones, each with a volume of $2304\pi \text{ cm}^3$.
13. Mount Fuji, the active volcano in Honshu, Japan, is 3776 m high and has a slope of approximately 30° . Mount Etna, in Sicily, is 3350 m high and approximately 50 km across the base. If you assume they both can be approximated by cones, which volcano is larger?



Mount Fuji is Japan's highest mountain. Legend claims that an earthquake created it.

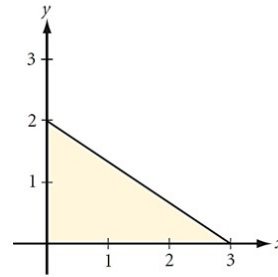
11.3 Exercises
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14. Bretislav has designed a crystal glass sculpture. Part of the piece is in the shape of a large regular pentagonal pyramid, shown at right. The apothem of the base measures 27.5 cm. How much will this part weigh if the glass he plans to use weighs 2.85 grams per cubic centimeter?
15. Jamala has designed a container that she claims will hold 50 in^3 . The net is shown at right. Check her calculations. What is the volume of the solid formed by this net? (h)



11.3 Exercises
pages 550 – 552

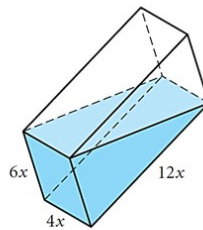
16. **Which is greater?** Which is greater, the volume of the solid formed by rotating the shaded figure at right about the x -axis or that formed by rotating about the y -axis? Explain.



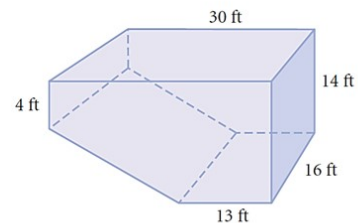
11.3 Exercises
pages 550 – 552

Review

17. Find the volume of the liquid in this right rectangular prism. All measurements are given in centimeters.



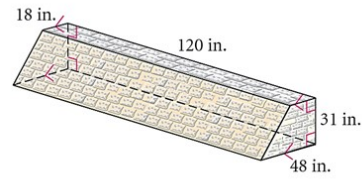
18. A swimming pool is in the shape of this prism. A cubic foot of water is about 7.5 gallons. How many gallons of water can the pool hold? If a pump is able to pump water into the pool at a rate of 15 gallons per minute, how long will it take to fill the pool? (h)



11.3 Exercises

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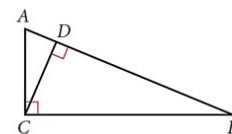
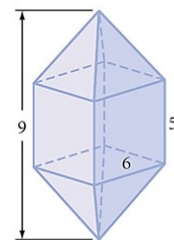
19. A landscape architect is building a stone retaining wall, as sketched at right. How many cubic feet of stone will she need?
20. As bad as tanker oil spills are, they are only about 12% of the 3.5 million tons of oil that enters the oceans each year. The rest comes from routine tanker operations, sewage treatment plants' runoff, natural sources, and offshore oil rigs. One month's maintenance and routine operation of a single supertanker produces up to 17,000 gallons of oil sludge that gets into the ocean! If a cylindrical barrel is about 1.6 feet in diameter and 2.8 feet tall, how many barrels are needed to hold 17,000 gallons of oil sludge? Recall that a cubic foot of water is about 7.5 gallons.



11.3 Exercises

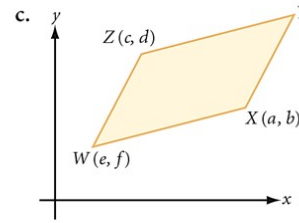
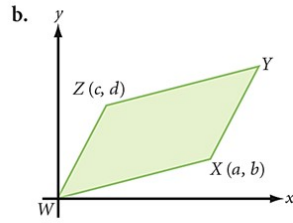
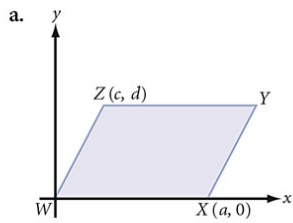
pages 550 – 552

21. Find the surface area of each of the following polyhedrons. Give *exact* answers.
- A regular tetrahedron with an edge of 4 cm
 - A regular hexahedron with an edge of 4 cm
 - A regular icosahedron with an edge of 4 cm
 - The dodecahedron shown at right, made of four congruent rectangles and eight congruent triangles
22. Given the triangle at right, reflect D over \overline{AC} to D' . Then reflect D over \overline{BC} to D'' . Explain why D' , C , D'' are collinear.



11.3 Exercises
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23. In each diagram, WXYZ is a parallelogram. Find the coordinates of Y.

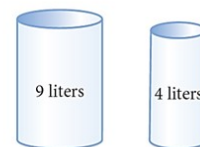


11.3 Exercises
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DEVELOPING MATHEMATICAL REASONING

Container Problem II

You have an unmarked 9 liter container, an unmarked 4 liter container, and an unlimited supply of water. In table, symbol, or paragraph form, describe how you might end up with exactly 3 liters in one of the containers.

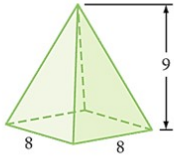


ANSWERS

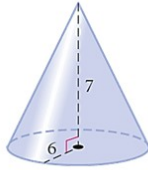
11.3 Exercises

Find the volume of each solid named in Exercises 1–6. All measurements are in centimeters.

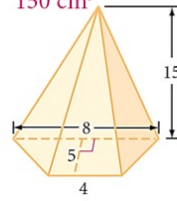
1. Square pyramid 192 cm^3



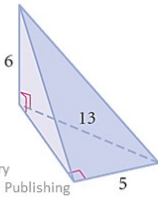
2. Cone $84\pi \text{ cm}^3 \approx 263.9 \text{ cm}^3$



3. Trapezoidal pyramid 150 cm^3

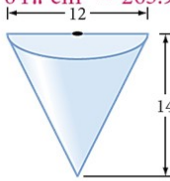


4. Triangular pyramid 60 cm^3

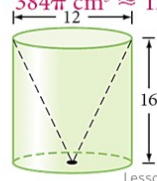


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5. Semicircular cone $84\pi \text{ cm}^3 \approx 263.9 \text{ cm}^3$



6. Cylinder with cone removed $384\pi \text{ cm}^3 \approx 1206 \text{ cm}^3$



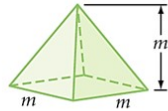
Lesson 11.3 Volume of Pyramids and Cones

ANSWERS

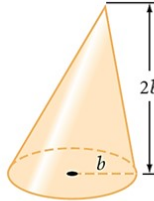
11.3 Exercises

In Exercises 7–9, express the total volume of each solid. In Exercise 9, what percentage of the volume is filled with the liquid? All measurements are in centimeters.

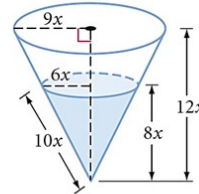
7. Square pyramid $\frac{m^3}{3} \text{ cm}^3$



8. Cone $\frac{2}{3}\pi b^3 \text{ cm}^3$



9. Cone $324\pi x^3 \text{ cm}^3$; 29.6%



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Lesson 11.3 Volume of Pyramids and Cones

ANSWERS

11.3 Exercises

10. Use the information about the base and height of each solid to find the volume. All measurements are given in centimeters.

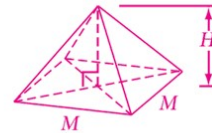
Information about base of solid	Height of solid	Triangular pyramid	Rectangular pyramid	Trapezoidal pyramid	Cone
$b = 6, b_2 = 7, h = 6, r = 3$	$H = 20$	a. $V = \frac{1}{3}bh = 120 \text{ cm}^3$	d. $V = 240 \text{ cm}^3$	g. $V = 260 \text{ cm}^3$	j. $V = 60\pi \text{ cm}^3$
$b = 9, b_2 = 22, h = 8, r = 6$	$H = 20$	b. $V = 240 \text{ cm}^3$	e. $V = 480 \text{ cm}^3$	h. $V = \frac{2480}{3} \text{ cm}^3$	k. $V = 240\pi \text{ cm}^3$
$b = 13, b_2 = 29, h = 17, r = 8$	$H = 24$	c. $V = 884 \text{ cm}^3$	f. $V = 1768 \text{ cm}^3$	i. $V = 2856 \text{ cm}^3$	l. $V = 512\pi \text{ cm}^3$

ANSWERS

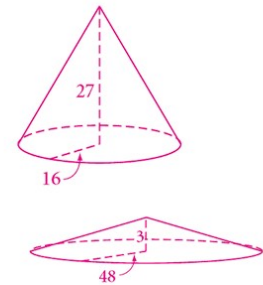
11.3 Exercises

11. Sketch and label a square pyramid with height H feet and each side of the base M feet. The altitude meets the square base at the intersection of the two diagonals. Find the volume in terms of H and M .
12. Sketch and label two different circular cones, each with a volume of $2304\pi \text{ cm}^3$.
13. Mount Fuji, the active volcano in Honshu, Japan, is 3776 m high and has a slope of approximately 30° . Mount Etna, in Sicily, is 3350 m high and approximately 50 km across the base. If you assume they both can be approximated by cones, which volcano is larger?

11. $V = \frac{1}{3}M^2H \text{ ft}^3$



12. sample answer:



Mount Etna is larger. The volume for Mount Etna is approximately 2193 km^3 , and the volume for Mount Fuji is approximately 169 km^3 .

ANSWERS

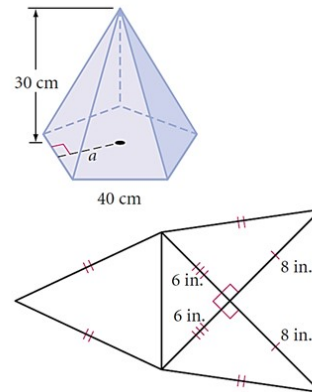
11.3 Exercises

14. Bretislav has designed a crystal glass sculpture. Part of the piece is in the shape of a large regular pentagonal pyramid, shown at right. The apothem of the base measures 27.5 cm. How much will this part weigh if the glass he plans to use weighs 2.85 grams per cubic centimeter?

78,375 grams

15. Jamala has designed a container that she claims will hold 50 in^3 . The net is shown at right. Check her calculations. What is the volume of the solid formed by this net? *h*

48 in^3



ANSWERS

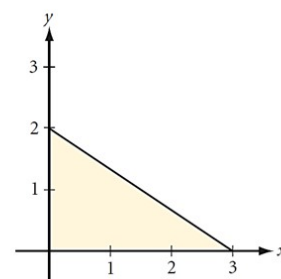
11.3 Exercises

16. *Which is greater?* Which is greater, the volume of the solid formed by rotating the shaded figure at right about the x -axis or that formed by rotating about the y -axis? Explain.

Rotating around the x -axis

Rotating around the x -axis generates a cone with height 2 and radius of the base 3, so volume is 6π .

Rotating around the y -axis generates a cone with height 3 and radius of the base 2, so volume is 4π .

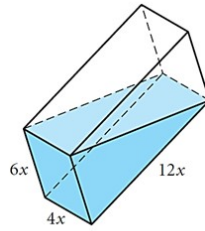


ANSWERS

11.3 Exercises

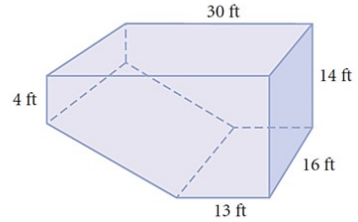
Review

17. Find the volume of the liquid in this right rectangular prism. All measurements are given in centimeters.



$144x^3 \text{ cm}^3$

18. A swimming pool is in the shape of this prism. A cubic foot of water is about 7.5 gallons. How many gallons of water can the pool hold? If a pump is able to pump water into the pool at a rate of 15 gallons per minute, how long will it take to fill the pool? h

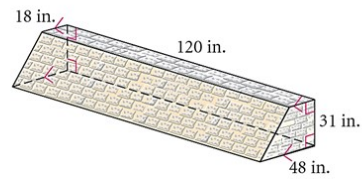


40,200 gal; 44 hr 40 min

ANSWERS

11.3 Exercises

19. A landscape architect is building a stone retaining wall, as sketched at right. How many cubic feet of stone will she need? 71 ft^3



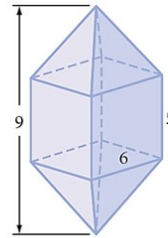
20. As bad as tanker oil spills are, they are only about 12% of the 3.5 million tons of oil that enters the oceans each year. The rest comes from routine tanker operations, sewage treatment plants' runoff, natural sources, and offshore oil rigs. One month's maintenance and routine operation of a single supertanker produces up to 17,000 gallons of oil sludge that gets into the ocean! If a cylindrical barrel is about 1.6 feet in diameter and 2.8 feet tall, how many barrels are needed to hold 17,000 gallons of oil sludge? Recall that a cubic foot of water is about 7.5 gallons.

403 barrels

ANSWERS

11.3 Exercises

21. Find the surface area of each of the following polyhedrons. Give *exact* answers.
- A regular tetrahedron with an edge of 4 cm
 - A regular hexahedron with an edge of 4 cm
 - A regular icosahedron with an edge of 4 cm
 - The dodecahedron shown at right, made of four congruent rectangles and eight congruent triangles

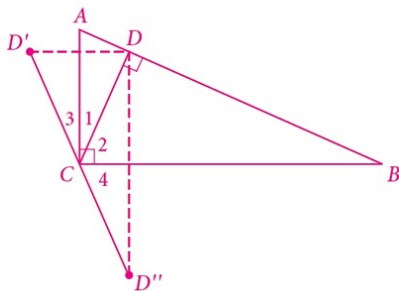
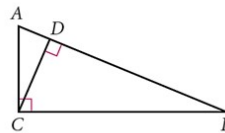


- $16\sqrt{3} \text{ cm}^2$
- 96 cm^2
- $80\sqrt{3} \text{ cm}^2$
- $(24\sqrt{13} + 120) \text{ cm}^2$

ANSWERS

11.3 Exercises

22. Given the triangle at right, reflect D over \overline{AC} to D' . Then reflect D over \overline{BC} to D'' . Explain why D', C, D'' are collinear.



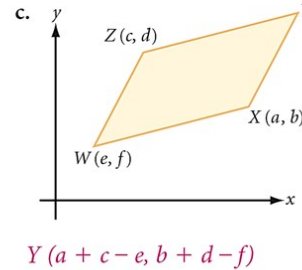
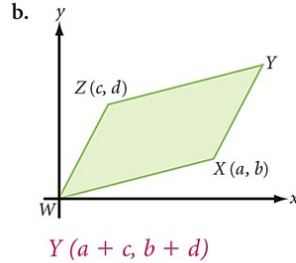
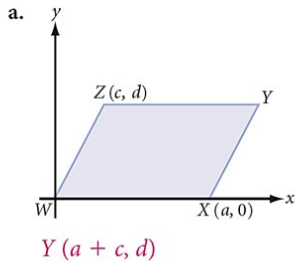
Possible answer:

From the properties of reflection, $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$.
 $m\angle 1 + m\angle 2 = 90^\circ$, so $m\angle 3 + m\angle 4 = 90^\circ$, and
 $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180^\circ$.
 Therefore $D', C,$ and D'' are collinear.

ANSWERS

11.3 Exercises

23. In each diagram, WXYZ is a parallelogram. Find the coordinates of Y.



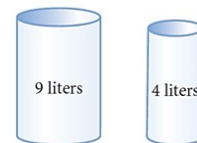
ANSWERS

11.3 Exercises

DEVELOPING MATHEMATICAL REASONING

Container Problem II

You have an unmarked 9 liter container, an unmarked 4 liter container, and an unlimited supply of water. In table, symbol, or paragraph form, describe how you might end up with exactly 3 liters in one of the containers.



One of many possible solutions:

Fill the 4-liter container. Dump into the 9-liter container. Repeat. Then fill the 4-liter container and dump into the 9-liter container until the 9-liter container is full (1 liter). Three liters will be left in the 4-liter container.