click for movie

## Warm-up:

- Watch the movie and decide:
  - > How long will it take the tank to fill up?
  - > Guess as close as you can.
  - > Give an answer you know is too high.
  - > Give an answer you know is too low.

What information will you need to know to solve the problem?

So how long did it take? How accurate was your guess?

click for movie

LESSON 11.2

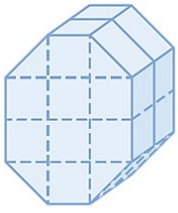
# Volume of Prisms and Cylinders

**Volume** is the measure of the amount of space contained in a solid. You use cubic units to measure volume: cubic inches ($in^3$), cubic feet ($ft^3$), cubic yards ($yd^3$), cubic centimeters ($cm^3$), cubic meters ($m^3$), and so on. The volume of an object is the number of unit cubes that completely fill the space within the object.




Length: 1 unit


Volume: 1 cubic unit



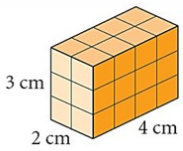
Volume: 20 cubic units

## INVESTIGATION

### The Volume Formula for Prisms and Cylinders

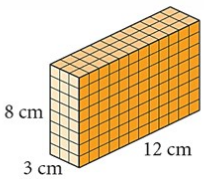
**Step 1** Find the volume of each right rectangular prism below in cubic centimeters. That is, how many cubes measuring 1 cm on each edge will fit into each solid? Within your group, discuss different strategies for finding each volume. How could you find the volume of any right rectangular prism?

**a.**



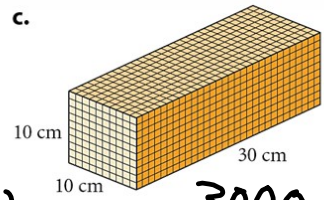

$24 cm^3$

**b.**




$288 cm^2$

**c.**




$3000 cm^3$

Notice that the number of cubes resting on the base equals the number of square units in the area of the base. The number of layers of cubes equals the number of units in the height of the prism. In a sense, you can visualize a repeated vertical translation of the base. So you can use the area of the base and the height of the prism to calculate the volume.



**Step 2** Complete the conjecture.

**Rectangular Prism Volume Conjecture** **C-92a**

If $B$ is the area of the base of a right rectangular prism and $H$ is the height of the solid, then the formula for the volume is $V = BH$.

In Chapter 8, you discovered that you can reshape parallelograms, triangles, trapezoids, and circles into rectangles to find their area. You can use the same method to find the areas of bases that have these shapes. Again visualizing a repeated vertical translation of the base, you can multiply the area of the base by the height of the prism to find its volume. For example, to find the volume of a right triangular prism, find the area of the triangular base (the number of cubes resting on the base) and multiply it by the height (the number of layers of cubes).

So, you can extend the Rectangular Prism Volume Conjecture to all right prisms and right cylinders.

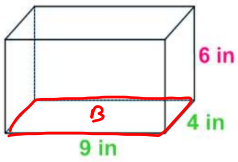
**Right Prism-Cylinder Volume Conjecture** **C-92b**

If $B$ is the area of the base of a right prism (or cylinder) and $H$ is the height of the solid, then the formula for the volume is $V = BH$.

**Find the volume of each prism.**

$V = l \times w \times h$

1.

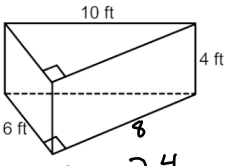


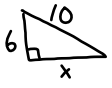

$B = 36$

$h = 6$

$V = 36 \cdot 6 = \boxed{216\ in^3}$

2.



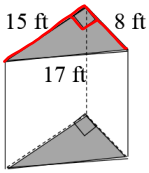

$6^2 + x^2 = 10^2$

$B = 24$

$h = 4$

$V = 24 \cdot 4 = \boxed{96\ ft^3}$

3. The prism below has a volume of 720 ft³. Find the height of the prism.




$720 = 60h$

$12 = h$

12ft

$B = \frac{15 \cdot 8}{2} = 60$

$h = ?$

The volume of a cylinder has the same formula as the volume of a prism, $V = Bh$. However, since the shape of the base of a cylinder is always the same we can be more specific with the formula.

What shape is the base of the ~~prism~~ cylinder and what is the area of that shape?

circle $B = \pi r^2$

If you substitute this area in for $B$ in $V = Bh$ you get the volume formula for a cylinder. What is the volume formula for a cylinder?

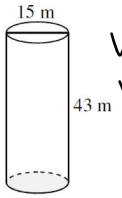
$V = \pi r^2 h$

**Volume of a Cylinder**

V = $\pi r^2 h$, where r = the radius of the circular base and h = height of the prism.

**Find the exact volume of each cylinder.**

4.




$V = \pi r^2 h$

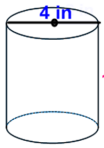
$V = \pi (7.5)^2 (43)$

$r = 7.5$

$h = 43$

Volume = $2418.75\pi \text{ m}^3$

5.




$V = Bh$

$V = 4\pi \cdot 10$

$V = 40\pi$

$r = 2$

$B = 4\pi$

$h = 10$

Volume = $40\pi \text{ in}^3$

6. The volume of a cylinder is $2160\pi$ ft$^3$ and has a height of 15 feet. Find the diameter of the cylinder's base.

h B

$$\frac{2160\pi}{15} = \frac{15\pi r^2}{15}$$

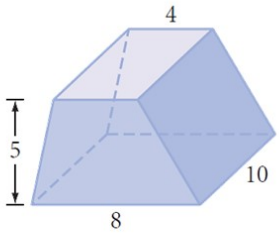
$$144 = r^2$$

$$12 = r$$

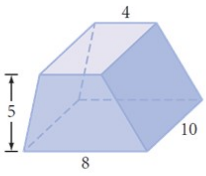
$$d = 24 \text{ ft}$$

**EXAMPLE A**

Find the volume of a right trapezoidal prism that has a height of 10 cm. The two bases of the trapezoid measure 4 cm and 8 cm, and its height is 5 cm.



*SOLUTION*




Find the area of the base.

$B = \frac{1}{2}h(b_1 + b_2)$ The base is a trapezoid, so use this formula to find the area of the base.

$B = \frac{1}{2}(5)(4 + 8) = 30$ Substitute the given values into the equation, then simplify.

Find the volume.

$V = BH$ The volume of a prism is equal to the area of its base multiplied by its height.

$V = (30)(10) = 300$ Substitute the calculated area and given height into the equation, then simplify.

The volume is 300 $cm^3$.





# Practice: 11.2 Volume of Prisms and Cylinders Practice

# #7 should look like this.

**Landscaping** Ingrid is building a shelter to protect her plants from freezing. She is planning to stretch plastic sheeting over the top and the ends of a frame. Which of the frames shown will require more plastic?